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Forced stick-slip oscillations allow the measurement of the friction force: Application to paper materials

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ABSTRACT

Different methods for measuring the friction forces are investigated in this paper. We consider the paper-on-paper contact as an example of application. We first underline several drawbacks for the two main standard methods, namely the inclined and horizontal plane methods. In particular, the horizontal plane test method often involves stick-slip oscillations that make the measurement of the friction force impossible. We then propose a method for characterizing these oscillations and removing their influence on the friction force measurement. The comparison of the proposed method to standards suggests that our proposed method delivers measurements that are much more accurate and repeatable. We finally discuss the validity of averaging the friction force measured during the sliding movement.
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1. Introduction

The friction phenomena being complex and not totally mastered, their study remains mainly empirical. The paper-on-paper contact [1] illustrates the study, but the results can be generalized to other contacts. The standards in the context of ISO, TAPPI, or AFNOR are based on the inclined and horizontal plane methods [2–9]. However, these methods suffer from major issues.

The inclined plane method consists in fixing one sample on a plane and the other on a weighted sled. The plane is then tilted from the horizontal to a critical angle, α, at which the sled starts to slide, as represented in Fig. 1a.

As the sliding starts, the friction force is called *break-away force*. The coefficient of static friction, μ_S, is then defined as the dimensionless ratio between the break-away force and the normal load, and characterized by the critical angle. The method is cheap, intuitive, and easy to run. However, we underline several drawbacks hereafter.

(i) The coefficient of static friction sometime depends on the normal load, e.g., in the case of polymers [10,11]. However, different coefficients of static friction measured with the inclined plane correspond to different normal loads. A comparison between different coefficients of friction is therefore impossible or requires multiple measurements with varying loads.

(ii) The repeatability of the measurement is poor ($\pm 2\%$) due to the difficulty to observe the beginning of the sliding. As we observe in our experiments, it is in particular the case for the rubber-on-steel contact, due to a large difference of stiffness of the materials [12].

(iii) As the sliding is not controlled, this method cannot be used to study the effect of repeated tests on a single couple of samples. This is a major drawback if the coefficient of static friction evolves with repeated slidings, as for example in the case of paper-on-paper contacts [13].

(iv) The method is useful to study only simple models of friction. In particular, the method does not allow the characterization of the kinetic friction and presliding displacements.

On the other hand, the horizontal plane method consists in measuring the pulling force, F_p, required to move the sled at constant speed, as represented in Fig. 1b. The maximum pulling force is considered as the break-away force and recorded to calculate the coefficient of static friction. Moreover, when the sled reaches the chosen velocity, the pulling force is averaged and considered as the *force of kinetic friction*. The coefficient of kinetic friction, μ_K, is then defined as the dimensionless ratio between the force of kinetic friction and the normal load. This method maintains the normal load constant and is adapted to the study of repeated slidings. However, we underline several drawbacks hereafter.

(i) The maximum pulling force does not necessarily correspond to the beginning of the sliding. For example, viscous friction may induce small displacements as the force of friction gets

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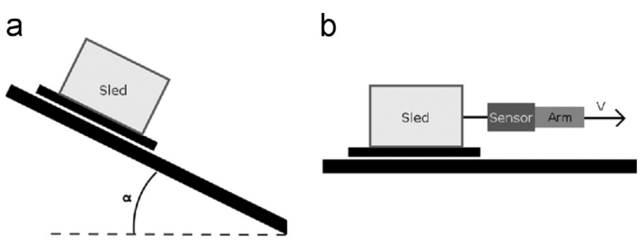
Nomenclature

x	position of the sled (m)
x_a	position of the arm (m)
l_0	spring elongation at rest (m)
u	spring elongation (m)
w	interpolation of the spring elongation (m)
k	stiffness of the sled–spring–sensor system ($N\,m^{-1}$)
F_p	pulling force (N)
F_f	force of friction (N)
m	mass of the sample–sled system (kg)
g	standard gravity ($m\,s^{-2}$)
F_N	normal load (N)
μ_S	coefficient of static friction (–)
μ_K	coefficient of kinetic friction (–)



Inclined plane Horizontal plane

Fig. 1. The standard methods for friction force measurement. (a) Inclined plane. (b) Horizontal plane.

established. We exemplify this problem in the rubber-on-steel case in the supplemental materials.

(ii) The acceleration is often neglected in the calculation of the friction force. Moreover, the determination of the maximal pulling force is based on a low number of measured points that decreases with the acceleration. As a consequence, the errors due to the acceleration of the sled increase with the acceleration. In particular, this situation is critical when the initial acceleration of the sled is produced by a shock between the force sensor and the sled, as described by several standards [3–5].

(iii) The method does not allow the separation of the force components due to static friction, kinetic friction, and mass-acceleration [2,14]. Avoiding stick-slip by increasing the stiffness of the sensor and/or the velocity of the displacement [15] would increase the sled acceleration and therefore the errors described in (ii). We remind that the stick-slip consists in a sequential build-up and release of stored energy in elastic components, resulting in cyclical acceleration and deceleration of the sled.

(iv) The method does not allow the measurement of micro-displacements which may be required for dynamic models of friction [16].

We underlined several drawbacks of the standard methods used to measure the friction force. These drawbacks limit the characterization of friction. Defining a protocol improving both the reproducibility and the range is therefore a major issue we propose to explore. We built a new experimental setup to conduct various experiments. The results are compared to standard methods. Advantages of the proposed setup and possible future improvements are finally discussed.

2. Materials and methods

2.1. Proposed setup

We use an horizontal plane tribometer in accordance with the standards (NF Q 03-082 [5] and TAPPI 549 [3]). The sled weights 837 g and its dimensions are 60 mm × 60 mm. The velocity of the arm is set to $5\,mm\,s^{-1}$. The proposed setup, called *oscillating setup*, consists in placing a spring between the force sensor and the sled. The sled–spring–sensor system has a constant spring stiffness, k (390 $N\,m^{-1}$). The spring induces a stick-slip phenomenon at roughly 2 Hz. We plug an analog filter to decrease the noise delivered by the force sensor. A Linear Variable Differential Transformer (LVDT) position sensor (accuracy ±0.01 mm) is placed between the sled and the arm, parallel to the spring, as represented in Fig. 2.

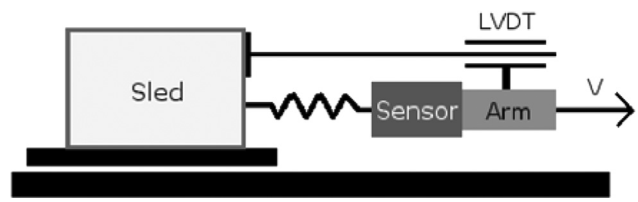


Fig. 2. Schematic view of the developed setup. A spring is placed between the sled and the force sensor. An LVDT measures the spring elongation.

The frequency of acquisition is 400 Hz. The measurements are processed using a Labview program.

2.2. Proposed method

The LVDT sensor measures the spring elongation, $u(x)$, defined as $u(x)=x_a-x-l_0$, where x_a, x, and l_0 represent the position of the arm, the position of the sled, and the spring elongation at rest, respectively. The fundamental principle of dynamics applied to the sled can be included in the expression of the coefficient of friction:

$$\mu=\frac{F_f}{F_N}=\frac{1}{m\cdot g}\left(F_p-m\cdot\frac{d^2x}{dt^2}\right)=\frac{k\cdot u}{m\cdot g}-\frac{1}{g}\left(\frac{d^2x_a}{dt^2}-\frac{d^2u}{dt^2}\right) \quad (1)$$

where F_f, F_p, m, g, and k represent the friction force, the pulling force applied by the arm on the sled, the mass of the sample–sled system, the standard gravity (g=9.81 $m\,s^{-2}$), and the spring stiffness of the spring, respectively. The second derivative of the elongation is noisy. Therefore, we identify a sixth order polynomial interpolation of the elongation for every measurement, as detailed in supplementary materials. The result of the interpolation is noted as $w(x)$. The difference between $w(x)$ and $u(x)$ is found to be lower than 0.1% which allows us replacing u by its estimation w. The arm is moving at a constant speed, its acceleration is therefore zero. We thus obtain from Eq. (1):

$$\mu=\frac{k\cdot w}{m\cdot g}+\frac{1}{g}\cdot\frac{d^2w}{dt^2} \quad (2)$$

In conclusion, the proposed method permits (i) the measurement of the pulling force applied by the arm on the sled, (ii) the calculation of the velocity of the sled, (iii) the calculation of the coefficients of friction, and (iv) the measurement of the acceleration of the sled.

2.3. Influence of stick-slip oscillations on the friction measured

The movement of the sled is different in the oscillating and horizontal plane setups. Consequently, the friction forces measured on both setups may be different, as reported in the literature [17]. The two main reported observations are (i) a change in time spent in static friction (also called *dwell time*), and (ii) a time delay between a change in velocity and the corresponding change in friction (also called *frictional lag*) [17,18]. On the one hand, the decrease in dwell time with stick-slip oscillations is known to reduce the coefficient of static friction. On the other hand, the frictional memory does not allow the proper characterization of the friction force during stick-slip oscillations, as the relative speed evolves quickly. These mechanisms are usually observed for low velocities and high spring stiffness [15].

However, these conditions are far from those involved in our experiment. Literature suggests that in the conditions considered in the oscillating setup, the two previously listed observations are not met for the paper-on-paper contact [14,15]. Consequently, the friction force can be properly characterized by the viscous model of friction [15]. This model consists in a linear increase in friction force with relative speed. This behavior can be characterized by our oscillating setup, as the relative velocity and the friction force are simultaneously measured.

2.4. Experiments

We study the friction of 80 g m^{-2} and 100 μm thickness writing papers. Roughness data of the paper are presented in supplemental materials. The relative humidity is 50% and the temperature 24 °C. The experiments are carried out on a length of 10 cm. For each experiment, ten pairs of samples are tested and the sliding is repeated five times. The results obtained from the oscillating setup are compared to results of the inclined and horizontal planes setups. These setups comply with the NF Q 03-083/TAPPI 548 and NF Q 03-082/TAPPI 549 standards. We emphasize that the same sled is used for the three setups, and the same tribometer is used as a basis of the horizontal plane and oscillating setups.

3. Results and discussion

3.1. An evolution of forces in three phases

Typical records for the paper-on-paper friction during one stick-slip-stick transition of the fifth repeated sliding are shown in Fig. 3.

We choose to consider the fifth repeated sliding as it is more stable than the previous slidings. Indeed, the force of friction of the paper-on-paper contact is known to decrease by up to −50% between the first and third repeated slidings [19,20]. Three phases (I, II, and III) and therefore two transitions (1 and 2) may be observed.

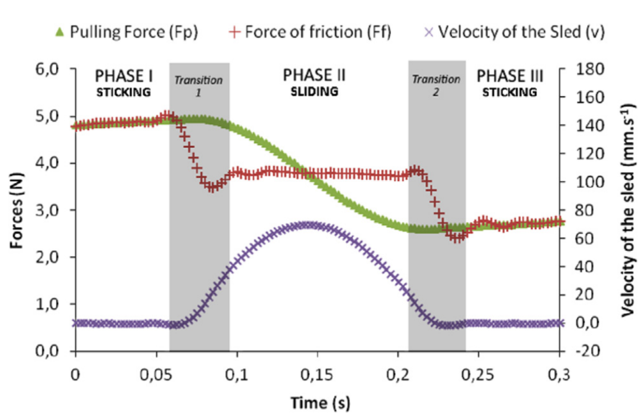


Fig. 3. Typical records obtained with the oscillating setup during one stick-slip oscillation of the fifth repeated sliding between two paper samples. The force of friction is calculated using Eq. (2).

- *Phase I and III* – The first and last phases consist in the sticking between the two samples. The phase starts as the two samples start sticking. The arm moves at a constant velocity, inducing a linear elongation of the spring and a linear increase of the force of static friction. The phase ends when the break-away force is reached and the sliding begins.
- *Phase II* – The second phase consists in the sliding between the two samples. During this phase, the ratio between the sled and arm velocities ranges from 0 to 10. This difference of velocity leads to a shortening of the spring and therefore a continuous decrease of the pulling force, F_p. On the other hand, the calculated friction force is nearly constant. Assimilating the pulling force to the force of friction is consequently not accurate in the sliding zone. The importance of the acceleration of the sled is underlined as we estimate the contribution of inertia to the pulling force, $m \cdot (\mathrm{d}^2x/\mathrm{d}t^2)$, to be as high as 25%.
- *Transitions 1 and 2* – The transitions are situated between the sticking and sliding phases. The force of friction thus evolves between the break-away force and the force of kinetic friction. We observe that the duration of the phase evolves with the number of measurement points used to calculate the interpolations of u, as described in the supplemental materials. The calculated force of friction is therefore mainly governed by the mathematical errors created by the calculation of interpolations. Additionally, the transition is also influenced by complex physical phenomena such as the so-called Stribeck effect [21] or the frictional lag [17].

In conclusion, our oscillating setup allows the measurement of both the break-away force and the force of kinetic friction. In addition, the method characterizes the sled position, velocity, and acceleration.

3.2. Comparison of the different methods

A sensitivity analysis of the different setups is summarized in Table 1.

We observe that the standard methods have a high uncertainty compared to the oscillating sled setup. We also compare the measurements obtained for the paper-on-paper friction with the inclined plane, the horizontal plane, and our oscillating setup. The results are summarized in Table 2.

We make the following observations:

- The oscillating setup and the inclined plane methods give similar dispersions of the coefficient of static friction (standard deviations of roughly 2%). The difference in their measurements is low (about 4%) and may partly correspond to the uncertainty of the inclined plan measurements (see Table 1).
- The coefficient of static friction measured with the horizontal plane differs by up to 9% from the one measured with the other methods, and is also more dispersed (standard deviations up to

Table 1
Sensitivity analysis on the measurement of the coefficients of static (μ_S) and kinetic (μ_K) friction during the first sliding and using the different setups.

Method	μ_S measurement	μ_K measurement
Inclined plan	0.72 ± 0.012 ($\pm 1.7\%$)	–
Horizontal plan	0.75 ± 0.020 ($\pm 2.6\%$)	0.58 ± 0.020 ($\pm 3.4\%$)
Oscillating sled	0.71 ± 0.0005 ($\pm 0.1\%$)	0.59 ± 0.0005 ($\pm 0.1\%$)

Table 2
Comparison of the coefficients of static (μ_S) and kinetic (μ_K) frictions obtained with different experimental setups. c_v represents the coefficient of variation of the measurement. Δref represents the gap between the reference, indicated by (ref), and the measurement. 10 experiments are carried out for each experiment.

Coefficient	Sliding	Inclined plane (± 0.012)			Horizontal plane (± 0.020)			Oscillating setup (± 0.0005)		
		Average	c_v	Δref	Average	c_v (%)	Δref	Average	c_v (%)	Δref (%)
μ_S	1	0.72	1.5%	(ref)	0.75	1.5	3.5	0.71	0.3	−2.2
	2	0.67	1.8%	(ref)	0.68	2.6	1.4	0.67	1.1	−0.5
	3	0.66	2.0%	(ref)	0.64	3.7	−3.9	0.65	1.1	−2.5
	4	0.66	1.8%	(ref)	0.62	3.6	−6.1	0.63	1.5	−4.5
	5	0.64	1.6%	(ref)	0.59	5.3	−9.1	0.62	1.5	−4.5
μ_K	1	–	–	–	0.58	3.5	(ref)	0.59	1.2	3.0
	2	–	–	–	0.53	3.7	(ref)	0.54	0.9	1.4
	3	–	–	–	0.51	3.1	(ref)	0.51	0.9	−0.2
	4	–	–	–	0.50	3.4	(ref)	0.50	0.8	0.0
	5	–	–	–	0.49	4.7	(ref)	0.49	0.9	0.2

5%). This result can be explained by the low frequency and noisy signal of the force sensor, as it is not totally removed by the analog filter. Increasing the filter level would however alter the peak measured at the beginning of the sliding and therefore is not implemented.

- The coefficient of kinetic friction is much more dispersed using the horizontal method than using the oscillating setup (standard deviations up to 5% and 1%, respectively). This horizontal plane defect is due to the apparition of stick-slip oscillations. Indeed, those oscillations are of the order of magnitude of the characteristic time of the analog filter, leading to a distorted signal.

The limits of the horizontal plane method are confirmed: the method delivers poor estimations of the coefficient of static friction and should not be used in case of macroscopic stick-slip movements. This macroscopic movement can be spotted when the standard deviation in pulling force measurements is greater than the standard deviation in friction force measured with the oscillating setup (roughly 1.5%). On the other hand, the oscillating setup appears to be adapted to the measurement of the coefficients of both static and kinetic friction. The oscillating setup appears to be adapted to the measurement of the coefficients of both static and kinetic friction. We attribute the measurement variations to the properties of the studied contact. On the other hand, the limits of the horizontal plane method are confirmed. Indeed, the method delivers poor estimations of the coefficient of static friction and should not be used in case of macroscopic stick-slip movements. This macroscopic movement can be spotted when the standard deviation in pulling force measurements is greater than the standard deviation in friction force measured with the oscillating setup (roughly 1.5%).

3.3. On the validity of averaging the friction force

During the phase II, see Fig. 3, the force of kinetic friction is constant. In the studied conditions, the paper-on-paper kinetic friction can thus be described by Coulomb's law of friction. In this situation, the sled oscillates around an equilibrium position, which evolves with the arm displacement. Consequently, the average acceleration of the sled is zero during the sliding. We therefore obtain an approximation of the coefficient of kinetic friction:

$$\mu_K = \frac{F_f}{F_N} \approx \frac{\overline{F_f}}{F_N} = \frac{k \cdot \overline{u}}{m \cdot g} + \frac{1}{g}\frac{d^2\overline{u}}{dt^2} \approx \frac{k \cdot \overline{u}}{m \cdot g} = \frac{\overline{F_p}}{m \cdot g} \tag{3}$$

where $\overline{*}$ represents the mean value of the quantity $*$. Moreover, the average pulling force during the sticking phases is equal to the force of kinetic friction, as the sled oscillates around an equilibrium position. Eq. (3) can thus be extended to the whole stick-slip oscillations. Eqs. (3) and (2) lead to similar results (differ by less than 1%). The approximation proposed in Eq. (3) is however much easier to use, as it consists in averaging the pulling force measured.

Conversely, the approximation gives poor results with the horizontal plane, as indicated in Table 2. According to Johansson et al., the high force components associated to accelerations, decelerations, and static friction are supposed to be responsible for this low accuracy [14]. But Eq. (3) shows that these components are canceled when averaging the pulling force. Thus, we rather suggest high-frequency stick-slip oscillations (above 100 Hz with tribometer we used) to be associated with a limited definition of the force measurement. For example, the stick-slip frequency observed on the horizontal plane is of the order of magnitude of the characteristic time of the analog filter we use. In this situation, the measured pulling force becomes inaccurate (in particular its extremes) and so would be the approximation proposed in Eq. (3).

In conclusion, we suggest the measurements obtained with the horizontal plane to be rejected in the case of stick-slip oscillations with periods that are similar to the sampling rate of the sensor. In the case of oscillations of lower frequency, the force of kinetic friction may be approximated by averaging the pulling force.

4. Conclusion and perspectives

The two studied standard methods for the measurement of the friction forces between paper materials (the inclined and horizontal plane methods) are limited and suffer from severe drawbacks. We have shown that the horizontal plane method can be improved to get an oscillating setup, by (i) placing a spring between the arm and the sled and (ii) using a position sensor measuring the spring elongation. Such a modification induces a controlled stick-slip movement. The proposed method gives lower dispersions and better accuracies for both the coefficients of static and kinetic friction.

Several improvements should be investigated in future works: (i) a position sensor permitting the study of micro-displacements would allow the use of more complete models of friction, (ii) the varying velocities of the sled during the stick-slip movement allow the study of their influence on friction, and (iii) the transient phases between the static and kinetic friction should be reduced by either replacing the LVDT by a velocity sensor, or by improving the mathematical calculation of the sled acceleration.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.triboint.2015.06.021.

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