NOTE

About the Theoretical Aspect of Multiple Light Scattering: Silvy's Theory

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Abstract: The general problem of light scattering in turbid media has been studied often. Compared to the exact solution for plane-parallel diffusers, the Kubelka and Munk solution appears as an approximation. The photon diffusion model is also an approximate solution restricted to the case of low light absorption, for which the prior work of Silvy has been ignored. It gives interesting relations between Kubelka-Munk conventional parameters and the more intrinsic quantities related to the properties of the media introduced by the photon model. © 2003 Wiley Periodicals, Inc. Col Res Appl, 28, 227–228, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.10149

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Numerous studies have been dedicated to light scattering in turbid media. For instance, the work of Kubelka and Munk[1,2] is very likely most often used in applied technical studies. Other approaches have been developed. However, they often consider light transmission as a two-flux propagation. A system of two differential equations is therefore obtained and solved using appropriate boundary conditions. However, it should be emphasized that this is only an approximate solution of this light scattering problem.

The general problem of radiative transfer has been studied by Chandrasekhar,[3] and exact solutions for plane-parallel diffusers, using the scattering and the absorbing characteristics of the materials, have been published by Giovanelli.[4] Compared to the exact solution, the Kubelka-Munk solution[1] appears as an approximation of practical interest in many industrial problems, mainly due to its simplicity. However, this solution makes use of K and S coefficients for the absorbing and scattering properties of the turbid materials, respectively. It has been shown, for instance by Gate[6,7] and Phillips and Billmeyer[8] between 1971 and 1978 that these coefficients are merely conventional parameters, rather than physical quantities related to the intrinsic properties of the media. This is an important drawback of the Kubelka and Munk theory. However, the prior work of Silvy,[5] and correlated discussions published in relation to his work about the photon theory, have been ignored.

An interesting approach of this scattering problem is the model based on photon diffusion, which, according to the words of Gate,[7] is "an approximate solution to the radiative transfer equation that is restricted to the case of relatively low light absorption." This restriction remains a minor point for most industrial problems and also concerns the case of the Kubelka-Munk analysis. Adopting the photonic point of view, the energy exchanges between the radiation and the matter take the form of absorption and scattering of photons. Therefore, it is necessary to introduce the specific efficient sections of absorption Σ_a and of scattering Σ_s, which are well-defined quantities, related to the intrinsic properties of

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the material. In comparison K and S may only be defined as follow:

$$S = \lim_{x\to 0} (dR_o/dx) \quad (1)$$

and

$$K + S = -\lim_{x\to 0} (dT_o/dx) \quad (2)$$

where dx, dR_o and dT_o represent infinitesimal variations of the thickness, reflectance and transmission factors, respectively.

The relations between (K, S) and (Σ_a Σ_s), were established by Silvy[5]

$$K = 2\sum\nolimits_a \quad (3)$$

$$S = 3/4\sum\nolimits_s - \sum\nolimits_a \quad (4)$$

These relations may be rewritten as follow:

$$\sum\nolimits_a = K/2 \quad (5)$$

$$\sum\nolimits_s = 4/3(S + K/2) \quad (6)$$

These relations have been also given later by Gate.[7] It is important to notice that S coefficient does not evaluate a true scattering characteristic of the material, but is dependent also on the specific efficient section of absorption Σ_a of the material.

In conclusion, we would like to draw attention to the fact that specific efficient sections of absorption Σ_a and of scattering Σ_s should receive more attention in the use of simple approximate light scattering theory. Their properties more clearly relate to those of the material used, and to their ability to replace K and S constants. We would like also to call attention to the prior work of J. Silvy in this field.

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