Abstract

Throughout the life of the track, ballast grains wear as a result of both the trains passing over the track and the maintenance operations (tamping). The morphology of the grains is changed: they lose angularity and fine particles are produced, causing a gradual loss of track performance. Eventually ballast substitution becomes necessary. In order to predict the evolution of this degradation, a multi-scale study is proposed. A comparison between discrete element method (DEM) simulations of Micro-Deval attrition test and of the passage of trains over a sleeper is performed in order to link the microscopic loading to the production of fine particles experimentally observed. The same numerical approach is also used with the simulations of the track to detect families of contacts in order to determine characteristic loading paths at the micro-scale. Loads and displacements of both systems are then compared in order to discuss the relevance of the Micro-Deval test.

Keywords: ballast grains, DEM, wear, Micro-Deval, multi-scale study, microscopic loading.

1 Introduction

After some years of service of the first high-speed railway lines in France (HSL), ballast has proven not to be resistant enough: grains wear faster than expected due to both the traffic of trains at high speed and the accumulation of maintenance operations (tamping), more frequent in this kind of lines. Ballast replacement has therefore been required much before than its originally expected lifespan.

The performance of ballast, as a thin layer of coarse grains, strongly depends on the shape, size and mineralogical nature of the grains composing it. Thus, in order to achieve the goal of maximizing ballast lifespan in future tracks, it is necessary to obtain a predictive model of ballast wear when it is subjected to complex loads.

Figure 1: Ballast grains after Micro-Deval attrition test, fine particles produced can be seen as dirt over the grains

Under the combination of both the dynamic stress imposed by the circulation of trains and tamping operations, ballast is gradually worn by fragmentation of grains and attrition of the grains surface at the contacts locus. The direct consequence of this degradation is the evolution of grain size and shape: the grading curve is moved towards the fine particles (d < 0.5 mm) (Figure 1) and the grains progressively lose their angularity (Figure 2). Eventually the cumulated wear will no longer allow ballast to perform properly: the internal friction angle is reduced limiting both the anchorage of sleepers and the transfer of loads to the platform. In addition, the presence in excess of fine particles renders tamping ineffective (fast evolution of track defaults after tamping) and reduces the permeability of the track.

Figure 2: New and artificially eroded (Micro-Deval) ballast grains

Attrition at the contact between two solid surfaces, a.k.a. third body problem, is a common field of tribology interest. One of the main assumptions states that the mass flux of generated fine particles strongly depends on the loading conditions. The classically used Archard’s model [1], presented in Equation (1), assumes that the generated volume of wear (W), i.e. the volume of fine particles produced, is proportional to both the normal force (f_n) and the relative displacement between the surfaces (s), which can be directly related to the work produced by friction forces. Although the model was conceived for flat surfaces, it has been widely used even for discrete elements simulations in granular media [2,3]. This model will be used as a basic tool to get a first approach to the wear quantification. However, it will be necessary to validate and calibrate the model based on experiments before using it as a reliable predictive tool.

$$W = k\frac{f_n \cdot s}{H} \tag{1}$$

Wear is then a micro-scale variable which depends on the macro-scale constraints to which ballast grains are subjected. Discrete elements simulations of ballast are thus proposed in order to assess the load at the contact scale, i.e. to undertake a scale change moving from the track scale to the contact one. However, due to calculation time, memory constraints, and the absence of good methods to simulate fracture and wear, numerical simulations cannot be precise enough when assessing micro-scale values: fine production and loss of angularity or roughness are not directly inferred from simulations, and must be therefore obtained from experimental data.

Empirically, in order to ensure a good performance of ballast against attrition, standard tests are performed in the quarry over some material samples to check and classify the quality of the extracted rock. Following the corresponding railway CE requirements, attrition resistance of ballast grains is experimentally assessed in a Micro-Deval device [4,5]. During this test, 10 kg of grains are turned at 100 rpm inside a 40 cm long drum with a 20 cm inner diameter during 140 minutes. The numerous contacts within the device, both between grains and with the steel drum, produce an accelerated wear process which causes changes in the morphology of ballast grains similar to those produced in the track. Due to its repeatability and wide use in the different European standards, Micro-Deval represents a good test to start with. It is easily reproducible both experimentally and numerically so it can be used as a link between both kinds of results.

Thus in order to search for the causes of degradation and quantify the production of fine particles, it is crucial to properly understand and compare the wear mechanisms involved in both systems: the Micro-Deval and the traffic of trains.

In this paper Micro-Deval test is simulated to link the microscopic loading path with the production of fine particles using Archard equation as a very first approach. Using the same numerical approach at the track scale, simulations of circulation of trains allow the determination of the microscopic loading in track conditions. Finally, both systems are compared in terms of loads, displacements and wear mechanisms in order to discuss the relevance of this test.

2 Methodology

The presented work is englobed in a larger project with the final objective of obtaining a predictive model of ballast wear as a function of the applied stress at the track scale.

Monitoring over time the production of fines and the evolution of the shape of ballast grains is not trivial at the track scale. Thus track conditions can only be simulated numerically to assess the contact loads and relative displacements between grains. On the other side, simulations cannot reproduce precisely and in an acceptable calculation time the micro-scale variables (changes in grains morphology, production of fines, etc.), which will be necessary to be retrieved from experimental tests. Hence some numerical and experimental campaigns have been proposed in different steps:

1. DEM simulations of both track conditions and reproducible tests (Micro-Deval): loading conditions are assessed and compared between systems, friction work is quantified as a first variable related to wear production, and families of contacts are searched in order to extract characteristic load paths that will be reproduced experimentally at a later stage.
2. Micro-Deval test is performed experimentally. The production of fines and the morphology of the grains are monitored over the time in order to follow their evolution. The shape is scanned using X-ray tomography while the surface roughness is assessed using a laser profilometer.
3. BCR3D, a shearing machine specially conceived to control each axis independently [6], is used to reproduce the characteristic contact force histories previously extracted from DEM simulations. Morphology and fines production are monitored in the same way as in the previous step.
4. Experimental data and data from DEM simulations are compared in order to create and calibrate a model of ballast wear production.

The results of the first phase of this 3-year research project are presented in the sequel.

3 DEM simulations

The software used to perform DEM simulations is LMGC90, an open source software which allows running 3D DEM models [7] with Non-smooth Contact Dynamics [8]. Polyhedral grains are randomly extracted from a database of 1000 different modelled ballast grains, built following the correct grain-size distribution of ballast and containing from 12 to 70 faces and from 8 to 37 vertices.

Below, the two different types of performed simulations are described in detail: Micro-Deval and train passage over a sleeper on a ballasted portion of track.

3.1 Micro-Deval

Micro-Deval attrition test has been numerically built following the European codes:

- The cylinder follows the geometric requirements with a 200 mm inner diameter and a length of 400 mm.
- The grains have been randomly chosen from the database and introduced inside the cylinder until the required mass of 10 kg has been reached. The mass has been calculated using the polyhedral volume and a density of 2700 kg/m^3 corresponding to granite.
- The cylinder turns at an angular velocity of 100 rpm.

In the case of the presented simulation, 109 grains have covered the required 10 kg of ballast. A friction coefficient of 0.8 has been used between ballast grains, and of 0.4 between grains and the steel drum (lateral walls included). Perfectly inelastic collisions are considered with a restitution coefficient equal to zero.

Since grains are considered as perfectly rigid bodies and no wear or crushing is considered, the system remains statistically the same along the time, so simulating the 140 minutes of the real test does not provide any extra information from that already provided by just some drum rotations at the steady state. Thus the simulation has been performed along 35.5 seconds (which corresponds to 59 rotations of the drum), with a time step of 10^{-4} s. Data has been written and analysed every 50 time steps, i.e. with a writing time step of $5 \cdot 10^{-3}$ s.

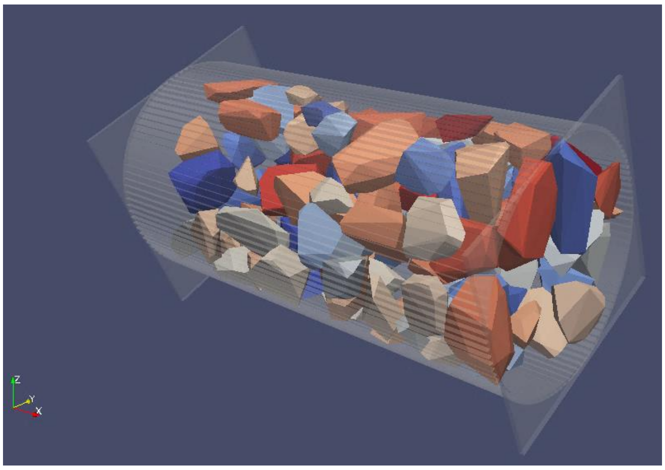

Figure 3: Snapshot of the Micro-Deval simulation. Ballast grains are coloured only for identification purposes

Two major aims have been followed with this simulation: to better understand what happens inside the drum (in terms of loads and particle trajectories), and to quantify the work made by friction forces.

3.2 Train passing over a sleeper

Searching for a compromise between calculation time and enough representative results, only half transversal section of a track has been considered to model the passage of a bogie over a sleeper. The section is limited longitudinally by two frictionless walls leaving a sample width of 1 m, and transversally by a frictionless wall in the inner side of the track (corresponding to the middle plane between rails) and a free boundary in the outer side. The floor is also considered as a rigid plane with a friction coefficient of 0.8. The corresponding half of a bi-block sleeper is located longitudinally centred. Figure 4 shows a snapshot of the simulation allowing the visualization of the described geometry.

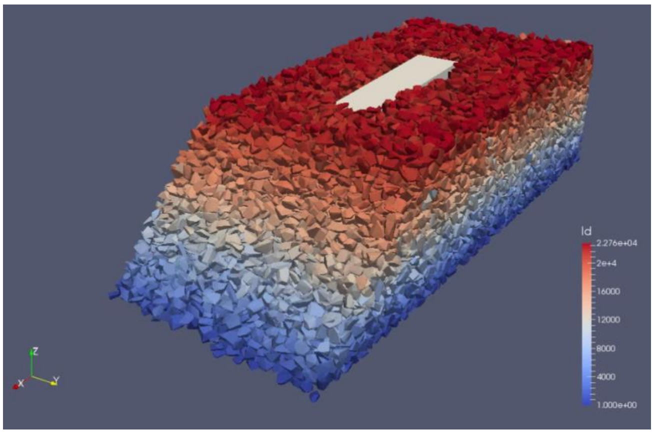

Figure 4: Snapshot of the track simulation. The limit walls are transparent to allow a proper visualization. Ballast grains are coloured only for identification purposes

A total of 22756 grains of ballast fill the considered section. The friction coefficient considered is 0.8, both between grains and between sleeper and grains. A double cyclic vertical force of 23 kN (each peak) is applied over the sleeper, corresponding each peak to the axle semiload (since only one rail is represented) of a HSL bogie at 300 kph (Figure 5).

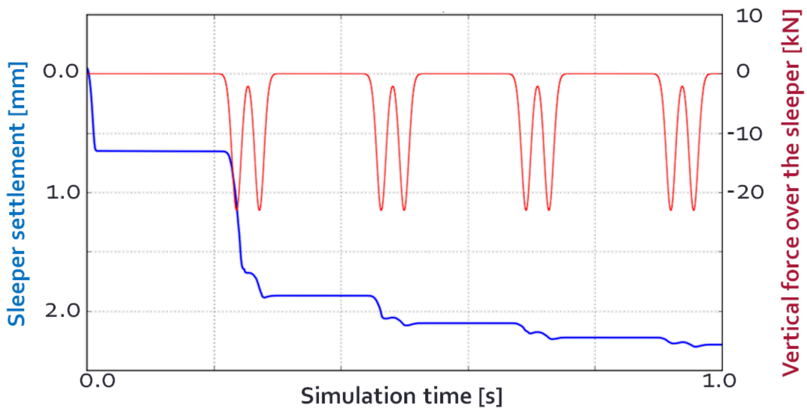

Figure 5: Settlement of the sleeper during the first four loading cycles

The simulation has been performed with a time step of $2 \cdot 10^{-4}$ s. Data has been written and analysed every 5 time steps, i.e. with a writing time step of 10^{-3} s.

4 Analysis

Following the different goals described, two main lines of analysis have been set:

- A global analysis of different contact quantities (density, normal force, relative displacement, etc.) is performed in order to understand the behaviour of each system and to detect the sources of grain wear. Following Archard's model, friction work is then calculated as a very first approach to assess qualitatively the production of fine particles. This line of analysis is applied to both simulations.
- A change in scale from track scale to contact scale is performed in order to search for families of contacts. Characteristic microscopic loading paths are then extracted for a future use in the BCR3D shearing device. The contacts inside the Micro-Deval device are too short so this line of analysis is only applied to the simulation of the track.

4.1 Sources of wear

In order to study the different magnitudes of the system at a global scale, a spatial distribution is proposed. To do that, the quantities are integrated along the simulation time inside every element of a fixed mesh. Thus the values are not linked to the grains but to the elements of the mesh, averaged among all the contacts located within in each time step.

4.1.1 Analysis of normal forces

The Micro-Deval test is a dynamic system in which grains turn very fast at almost the same speed as the cylinder (100 rpm). As shown in Figure 6, two different phases can be distinguished.

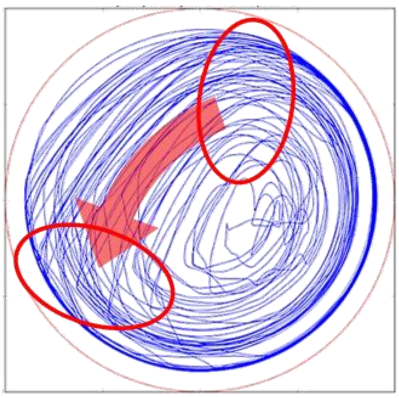

Figure 6: Trajectory of one grain during 35 s of Micro-Deval test

The grains first go upwards due to the centrifuge force and the friction between grains and cylinder. Due to the high density of grains in this first part of the trajectory, most of contacts are produced here. These contacts are quite stable along time but their intensity is low (with an average normal force of less than 2 N between ballast grains). In Figure 7, the plot of the cumulated normal force per turn (left) shows this accumulation of contacts in the lower-right side of the cylinder.

In a second phase, grains are unstuck from the cylinder due to gravity and they fall freely to the lower-left side of the cylinder, impacting against the steel drum or against other grains and starting again the upward movement. The impacts after this free-fall phase are much stronger (with contacts of more than 1 kN) but last for just one time step. In Figure 7, the average normal force in each local mesh element (right) shows how impacts on the left side are much stronger and irregular than those occurring during the upward phase.

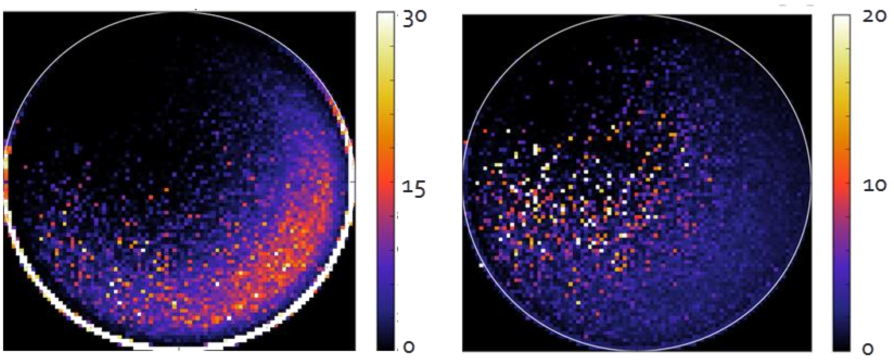

Figure 7: Cumulated normal force (left) and average normal force (right), in Newtons, along one Micro-Deval drum turn

The first phase would presumably produce slow but constant degradation while the second phase, much more violent, would produce fast degradation and eventually even grain crushing.

On the other side, the passage of a train is a much more static system and this difference between cumulated and average forces cannot be observed. Instead, the weight of the sleeper and of the train is transmitted to the platform through strong columns of grains, which absorb the majority of the efforts. In Figure 8 the average normal force along one loading cycle is plotted, showing how the efforts spread downwards in a triangular shape.

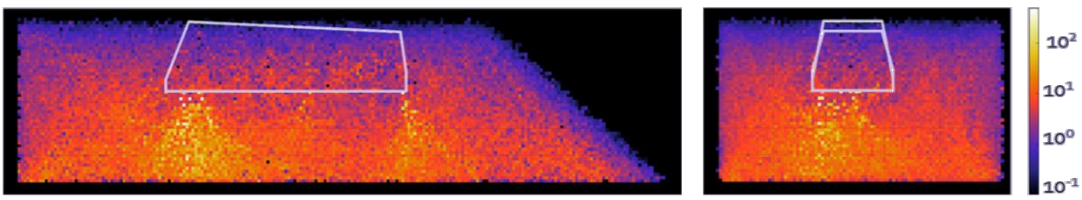

Figure 8: Average normal force, in Newtons, along one passage of a bogie

These chains of forces are created in specific places, being those grains located just under the sleeper the ones supporting the highest contact efforts.

If the whole sample is considered the variance of normal contact forces is high, which is understandable since many areas are almost not loaded (free boundaries) and other receive the majority of the efforts (under the sleeper). During one loading cycle, the average contact normal force is only 8 N, while only 1% of the contacts exceeds 75 N, and 0.1% surpasses 300 N. However, the maximum normal force rises to more than 12 kN.

Finally, if these results are compared to a relaxed phase, where no exterior force is applied, the highest efforts are the most affected. The maximum contact force in a relaxed phase is barely 1 kN, while only 0.1% of contacts exceeds 90 N. Nevertheless, the average is only reduced to 5 N, i.e. most of the ballast is actually inactive and only used as a dead mass.

4.1.2 Analysis of friction work

In order to detect the potential zones of wear and to perform a first qualitative analysis of wear production, Archard's model (Equation 1) is used. Work produced by friction forces, between two grains in contact, is assumed to be proportional to the product $f_n \cdot s$, i.e. between the contact normal force and the relative displacement between the grains.

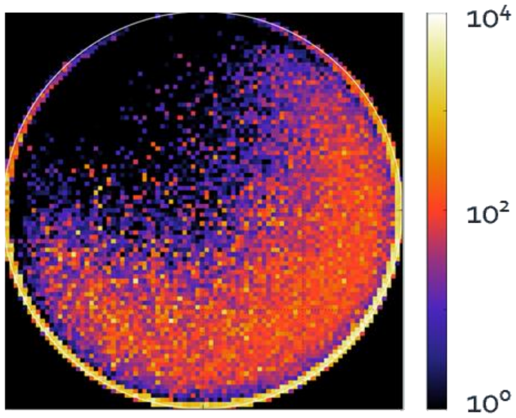

Figure 9: Friction work, in J / mm^2, along one Micro-Deval drum rotation

Figure 9 shows the spatial distribution of the cumulated friction work during a complete rotation of the Micro-Deval drum (0.6 seconds). The first thing that can be noticed is that impact zones (less but stronger contacts) develop the same order of magnitude of cumulated friction work than the dense zone (more but weaker contacts). In addition, it can be observed a significant amount of work appearing over the surface of the cylinder, which does not necessarily mean that the ballast-drum contacts are the main generators of wear. The space-fixed nature of the mesh causes a constant presence of contacts within the mesh elements over the drum surface, leading to a higher sum of friction work, i.e. there is not necessarily more work generated by these contacts but many more contacts in these specific mesh elements. On the other side, the two different materials (granite and steel) absorb contact energy in a different proportion (different speeds of degradation and heating) so it is not obvious to deal with them. The actual contribution of ballast-drum contacts in the Micro-Deval test should be studied carefully during the different experimental campaigns to be performed in a future phase of the project.

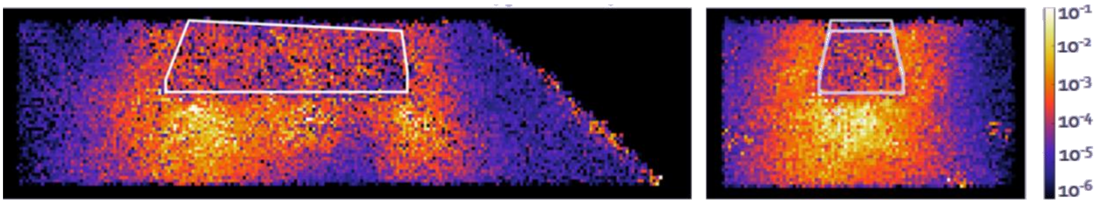

Figure 10: Friction work, in J / mm^2, along one passage of a bogie

The same plot in the track simulation (Figure 10), along one loading cycle (0.1 seconds), shows a higher potential wear all around the sleeper, and especially at the areas just below the sleeper. Some work is also observed over the slope in the free boundary, due to some grains rolling down. Comparing the order of magnitude of the work of both simulations, Micro-Deval shows a much higher wear potential than the passage of a bogie, which is not surprising since the attrition test is conceived for generating an accelerated degradation on the grains.

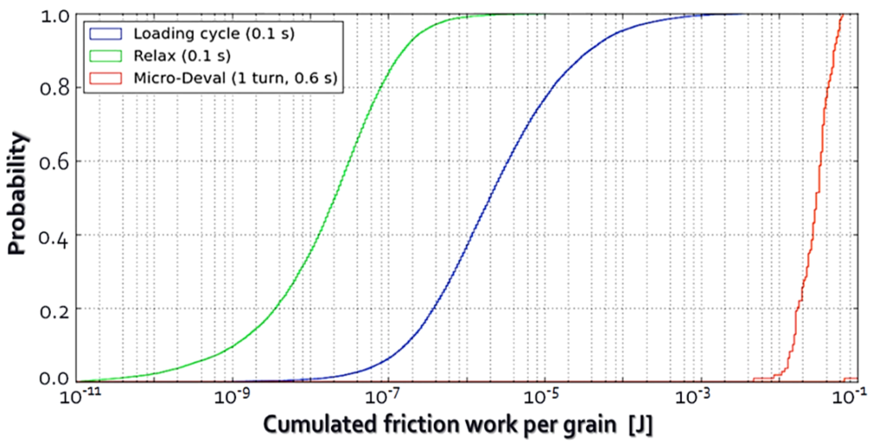

Figure 11: Cumulated normalized histogram of the friction work absorbed by each grain during a Micro-Deval rotation (red), the passage of a bogie (blue) and a relaxed phase (unloaded track) as a reference curve (green)

Figure 11 shows a cumulated normalized histogram of the work absorbed by each individual grain in both simulations. As observed in the plots above, a Micro-Deval turn (red) develops much more friction energy (more than five orders of magnitude) between ballast grains than the passage of a bogie (blue). Besides, the grains in the

Micro-Deval erode in a pretty homogeneous way, showing a much lower variance. Since the whole sample is considered, the variance in the track simulation is high, leading to only 1% of the grains absorbing more than 50% of the total friction work. Concerning the sleeper, it absorbs between 1% and 2% of the total work. Finally, even if the system is supposed to be absolutely static during the relaxed (unloaded track) phase (green) some work is generated, probably due to numerical issues.

In conclusion, Micro-Deval and track seem to have different mechanisms for degrading grains. Micro-Deval generates higher and more homogeneous friction work, which is directly related to friction wear. This degradation comes from friction between grains but also from impacts after the free fall phase and friction between the grains and the steel drum. These two last mechanisms are not very representative of what actually happens in the track.

On the other side, Micro-Deval degradation is homogeneous, not only because all grains wear more or less at the same speed, but also because each grain receives impacts all around its surface due to the dynamic nature of the test. Grains in the track have a very restricted movement, so they are in contact with the neighbour grains in approximately the same areas, causing a local mechanism of wear. This local wear will change the shape of the grain, eventually causing a rearrangement of grains and wear affecting other areas of the grain.

Even with all these differences, what has been observed in actual HSL is that the final morphology of ballast grains is very similar after a Micro-Deval test and after being in the track for a long period of time.

4.2 Microscopic loading

Understanding and then reproducing what happens at the contact between two grains is crucial to properly model the changes in morphology of ballast aggregates. Thus the second part of the DEM analysis consists in performing a scale change from the track scale to the contact scale. This analysis is carried out only in the simulation of the passage of a bogie.

The main goal of this study is to detect and classify, if they exist, different families of relatively permanent contacts that can be considered as characteristic, either due to their repetitiveness or to their importance in the wear process of the grains. These contacts will be then reproduced and studied, in a further stage of the project, using a shearing machine capable of controlling all three axes independently (BCR3D [6]).

During the first part of the study (chapter 4.1), contacts were considered at each time step independently, so remaining in time of contacts was not contemplated. On the contrary, this second part of the study needs contact remaining in time to be considered. To do so, each contact concerning two specific grains starts to count when it first appears, it is monitored at each time step (kind of contact, position, contact force, relative displacement, etc.) until there is a lack of it in one time step, so the previous time step will be considered the end of the contact. Thus if it appears in a later stage, it will be counted as a different contact, even if it concerns the same two grains. In this way, different quantities can be studied, such as the duration in time of the contacts (T), the cumulated displacement or trajectory of the contacts (D) and the effective displacement (δ), i.e. the distance between the final recorded position and the initial one.

In order to facilitate the geometric interpretation, only simple contacts are considered, i.e. those grains who share a unique point in common while in contact. Besides, since the goal is to detect relatively permanent contacts, contacts lasting for just one time step are not included.

4.2.1 Cumulated-effective displacement

When talking about contact trajectories, the local frame of each grain is considered, i.e. the trajectory will be the trace left by a contact on the surface of the grain (Figure 12). Thus vertex contacts are punctual so the local displacement should be equal to zero, despite some numerical issues but negligible in comparison (represented as a point in grain 1 of Figure 12 left). Edge contacts will stay in the line of the edge, although they can vibrate along the line (Figure 12 right). Finally, contacts over a face will generate a 2D trajectory over the surface plane (grain 2 of Figure 12 left). It is also possible, but very unlikely, to have a change in the contact face without creating a gap between the grains (loss of the contact), leading to a 3D trajectory.

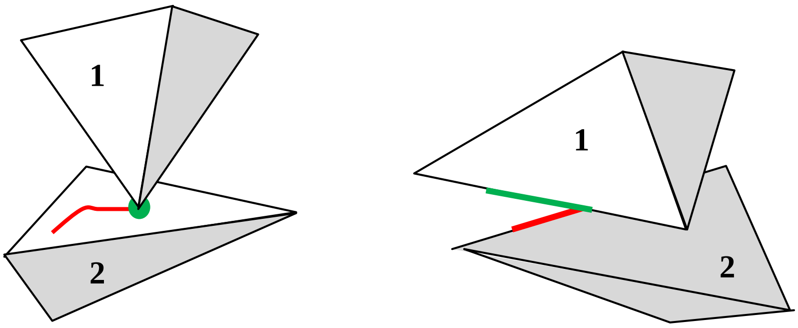

Figure 12: Trajectory in the local frame of each grain of vertex-surface contact (left) and edge-edge contact (right). In green, it is represented the trajectory in the local frame of grain 1, while in red in the local frame of grain 2

A cyclic loading like the one concerning the passage of a bogie can presumably generate two main families of contacts:

- Contacts with a relatively linear trajectory, in which cumulative (D) and effective (δ) displacements have similar values.
- Contacts with a high vibrating trajectory, in which D will be significantly higher than δ.

Figure 13 shows the D-δ plot zooming on the contacts with an effective displacement (δ) higher than 10^{-6} m, which represents only 0.6% of the contacts but more than 25% of the total work of the system. The colours represent the amount of work generated by the contact. Although contacts lasting for just one time step have not been considered, most of the contacts are still situated over the line D=δ or very close to it, i.e. mostly linear trajectories. As expected, contacts with higher displacements generally show a higher work generation. However further groups or families cannot be distinguished. One of the possible reasons could be the amount of calculating time and memory the computer needs both to simulate and to deal with the great amount of output data. Due to this, a maximum of two cycles of loading (of 13 cycles applied along the whole test) have been treated at the same time so far. A different approach considering only the areas of maximum work generation in order to reduce the amount of data is planned in future analysis. Nevertheless, the whole simulation lasts for only 6 seconds, not allowing big rearrangements of grains and restricting considerably this approach.

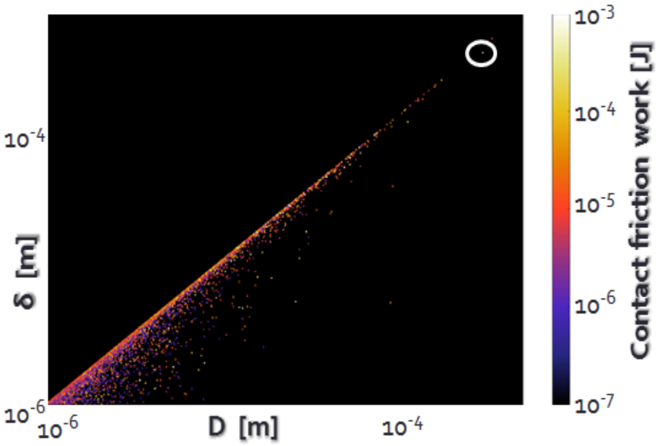

Figure 13: Cumulated (D)-effective (δ) displacement plot, zooming on the contacts with δ higher than 10^{-6} m. The friction work is shown as extra information. The contact object of study in section 4.2.2 is circled in white

4.2.2 Microscopic study of a contact

Even though this work is a very first attempt to classify contacts (failing to give the expected results), it is useful to detect some individual contacts that are worth studying. In particular, there is one contact (circled in Figure 13) that stands out among the others. This contact has one of the highest values in cumulated (0.19 mm) and effective (0.17 mm) displacements and work generated (10^{-3} J) and it lasts for 25 time steps (0.025 s) before the first interruption. Knowing that, a microscopic study has been performed on the contact between these two specific grains along the total duration of the two cycles of loading, with the observed pause in between (0.33 s in total).

The considered contact corresponds to a vertex-face contact between two grains located just below the sleeper, in the zone of maximum work according to the simulation (and also to the observations in a real track). Figure 14 shows the trajectory followed by the vertex over the face, i.e. the “scratch” left by one grain over the other. The trajectory is mostly linear and monotonic, i.e. without coming back, along a 0.5 mm path. It is also possible to distinguish the pause between the two loading cycles as a zone of little and vibrating movement.

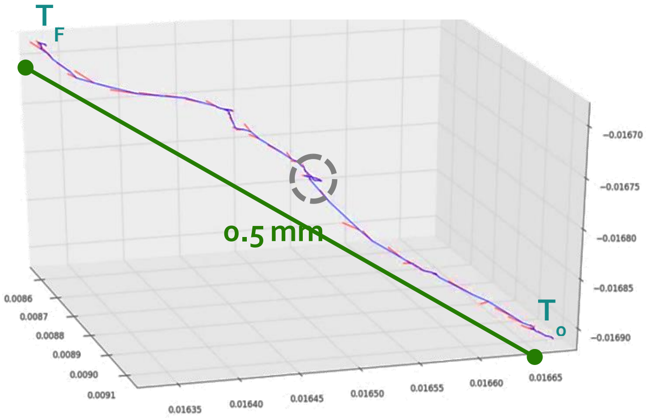

Figure 14: Trajectory of the contact over the face of the “scratched” grain. The stop between the two loading cycles has been circled in grey

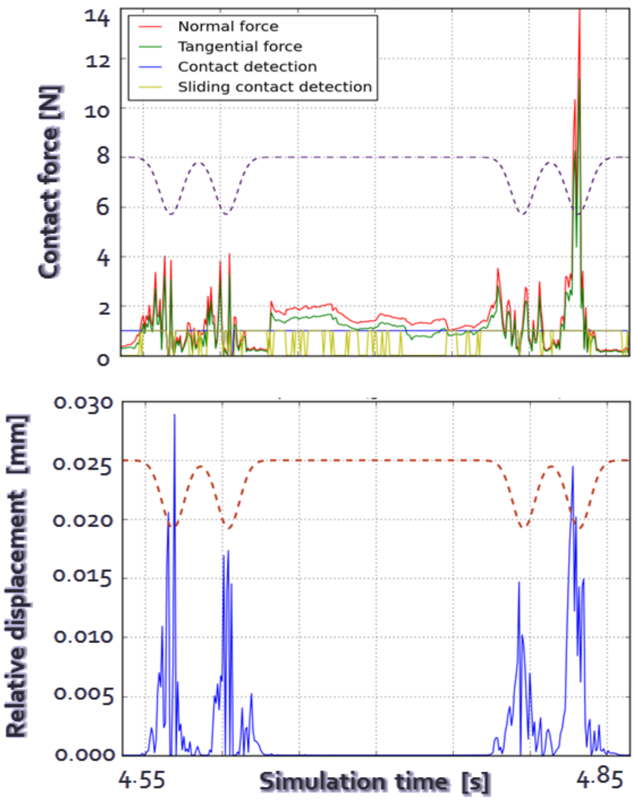

Figure 15: a) evolution of the normal (red) and tangential (green) contact forces, and sliding contact detection (yellow); b) the relative displacement between both grains throughout time. The cyclic loading is plotted with a dashed line as a reference

Figure 15 shows the evolution throughout the time of the normal and tangential contact forces (up) and of the relative displacement between both grains (down). As expected, contact forces during the loading phases are much more irregular and with higher peaks than in the relax phase. The relative displacement shows the same behaviour, achieving the greater values exactly at the peaks of the external cyclic load.

Finally, Figure 16 shows the cumulated amount of work absorbed by each grain along the two loading cycles, assuming that each grain absorbs 50 % of the contact energy. Following the results above and considering that the work is computed as the product between normal force and relative displacement, the evolution follows the same tendency as before, with the higher increases at the loading peaks.

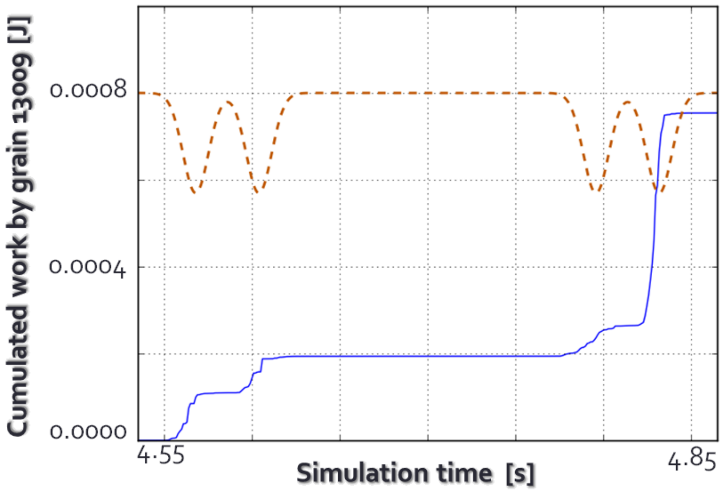

Figure 16: Evolution of the work absorbed by each grain only considering the described contact, the cyclic loading is plotted with a dashed line as a reference

5 Conclusions and perspectives

In this paper two DEM simulations have been presented, one corresponding to the Micro-Deval attrition test, and the other corresponding to the passage of a train over a sleeper at 300 km/h.

The analysis of normal forces in the systems has shown that the Micro-Deval test is a very dynamic system with two different mechanisms of wear: a slow but constant degradation in the upwards phase and a punctual but strong degradation coming from the impact after the free fall phase of the grains.

The same analysis on the track has confirmed the creation of strong chains of grains which support the majority of the load. These chains of forces and the grains surrounding them absorb almost all the energy of the system, being the zones of maximum degradation of ballast.

The analysis of the product $f_n \cdot s$, i.e. the work produced by friction forces, has confirmed the results above. In the Micro-Deval, the work generated in ballast-ballast contacts is distributed both in the upward movement and in the impact zone, with more or less the same order of magnitude. The contacts between ballast and cylinder generate a big amount of energy and should be carefully treated in a further study. In the track, the maximum work is generated all around the sleeper, especially below it, since the maximum forces and displacements occur in this zone.

The comparison between both systems has demonstrated that the Micro-Deval generates indeed a bigger amount of work per cycle than the track. However, this degradation comes not only from friction between ballast grains but also comes from other mechanisms that are not very representative of what actually happens in the track: impacts after the free fall phase and friction between the grains and the steel drum. In addition, grains in Micro-Deval degrade in a much more homogeneous way than in the track.

On the other side, the first attempt to detect some families of contacts in the track, using a cumulated-effective displacement analysis, has not yet provided the expected results: almost all the contacts lie in the line $D=\delta$, i.e. contacts with relatively linear trajectories. However, this analysis has been useful to detect some interesting individual contacts that have been therefore studied in order to understand what happens at the micro-scale.

A particular contact has been selected for an in-depth analysis due to its high relative displacement and generated work, corresponding to two grains lying just below the sleeper. As expected, relative displacements and normal forces reach the maximum peaks when the cyclic external force is maximal. Furthermore, the studied vertex-face contact generates a scratch of around 0.5 mm over the face throughout two loading cycles, always in a forward movement. This only-forward trajectory could be explained by the short time of study (passage of two bogies).

In a further step, other simulations should be performed like a tamping operation and other reproducible tests in order to enrich the available data. Moreover, other wear models should be investigated, especially those including wear produced by impact. Finally, the numerical results will be compared and linked with the experimental data retrieved form Micro-Deval tests and shearing tests with the BCR3D device.

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