Displacement fluctuations in granular materials: a direct manifestation of grain rearrangement

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ABSTRACT: Grain rearrangement, which is the basic mechanism of irreversible strain in granular materials, results in displacement fluctuations, i.e., the displacements of grains deviate from the value dictated by a continuum field. These fluctuations play a similar role as dislocations in crystals, in the sense that they are at the very origin of irreversible strain in granular materials. The analysis of fluctuations is a hot topic in granular physics and mechanics. However, to the best of the authors knowledge, it has been investigated so far only by means of numerical (DEM) simulations. We present herein an experimental study of displacement fluctuations in quasi-static deformation of a 2D granular material deformed in a shear apparatus named $1\gamma2\varepsilon$. This apparatus allows a specimen composed of an assembly of rods to be subjected to general 2D-loading conditions, by independently applying normal strain along the vertical and horizontal axes, as well as shear strain. The kinematics of the centers of each rod in the specimen are followed by means of a 2D Particle Image Tracking (PIT) technique, which is applied to a sequence of digital photographs acquired throughout the duration of a test. This experimental work presents clear evidence of spatial organization of fluctuations in vortex-like structures (fluctuation loops) that are observed for different sizes of the strain increment considered (referred to as strain window). For small strain windows, these vortex patterns are quite intermittent, whereas they become more persistent when the size of the strain window is increased. The analysis of the experimental data indicates the existence of a minimum length scale of the fluctuation loops.

1 INTRODUCTION

In a granular material, a macroscopically homogeneous deformation does not correspond to a homogeneous field of displacement gradient when looking at the individual grains. Due to geometrical constraints at the grain scale (mutual exclusion of grain volumes), grains are not able to displace as continuum mechanics dictates they should. This can be pictured by imagining an individual in a crowd of people who all wish to go to the same place: although the long-term displacement of each individual is equal to the displacement of the crowd, the steps of each individual are erratic. Classical continuum approaches disregard this feature, which is apparent only at "small" strains (*i.e.*, for displacements that are small with respect to the size of rigid grains, or the size of contact indentation for deformable grains). However, the deviation of a grain's displacement from the value dictated by the continuum field (referred to as *fluctuation*) is likely to hold valuable information about the characteristic length(s) involved in grains' rearrangement, which is the principal mechanism of irreversible deformation in granular materials. Displacement fluctuations, often spatially organized in the form of vortices, have been observed in quasistatic experiments only by Misra and Jiang 1997. Several numerical studies using Discrete Elements (DEM) report similar observations, e.g., Williams and Rege 1997, Kuhn 1999, Combe and Roux 2003, Tordesillas et al. 2008, Rechenmacher et al. 2011. Radjai and Roux 2002 investigated such fluctuations by making use of statistical analysis that is typically used in fluid dynamics. The main finding of Radjai and Roux 2002 was that displacement fluctuations in granular materials show scaling features with striking analogies to fluid turbulence. Inspired by the approach of Radjai and Roux 2002, the present study analyzes fluctuations measured from experiments on a 2D analogue granular material subjected to (quasistatic) shear.

2 SHEAR TEST

Shear tests have been performed in the $1\gamma2\varepsilon$ apparatus, which is essentially a plane stress version of the directional shear cell developed for testing soils (see Calvetti et al. 1997, Joer et al. 1992, Charalampidou et al. 2009 for details). Results from only one such test are discussed herein, although the results are consistent throughout the entire programme. In this experiment, an assembly of 2000 2D "grains" (wooden cylinders $6cm$ long) having four different diameters (8, 12, 16 and 20 mm), was slowly deformed in simple shear at constant vertical stress $\sigma_n = 50$ kPa, Fig. 1. Note that the contact properties among the (wooden) grains compare with the properties imposed in DEM simulations by Radjai and Roux 2002. The vertical sides of the enclosing frame are tilted up to $\gamma = 0.26$ $(15°)$ while the length of the horizontal sides is kept constant. The shear strain rate is 8.2×10^{-5} s^{-1}, which is small enough to ensure a quasi-static regime according to the inertial number criterion suggested by Combe and Roux 2003. A video of the test can be found at http://youtu.be/B4Dfesn5vhs.

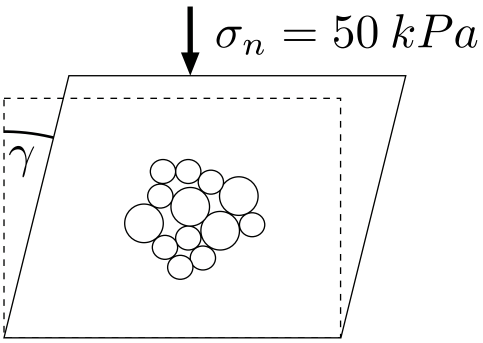

Figure 1: Sketch of a shear test in the $1\gamma 2\epsilon$ apparatus.

During the test, the normal (σ_n) and tangential (σ_t) stresses are measured at the boundary of the sample (shown in Richefeu et al. 2012). The global stress-strain response is typical of a dense 2D granular material, with a peak friction angle of about $26°$ at $\gamma \simeq 0.06$ and a dilatant behaviour throughout. A digital camera was used to acquire 24.5 Mpixels images every 5 seconds (*i.e.*, a shear strain increase of $\Delta\gamma \simeq 4 \times 10^{-4}$, called strain window in the following) thoughout the test.

The displacements of all the grains are measured by means of a software named TRACKER specially developed to process the digital images and measure (with sub-pixel resolution) the in-plane displacement and rotation of each individual grain from one image to another. This software uses a discrete grain-scale version of digital image correlation technique hereafter named PIT (Particle Image Tracking) that is well suited for tracking rigid particles. The principle of the PIT technique is detailed in Richefeu et al. 2012 and Combe and Richefeu 2013. From the 24.5 Mpixels images taken during the shear test, particles' displacements are assessed with a error that is less than 0.1 pixel Richefeu et al. 2012. As an example, Fig. 2 shows the displacements of all grains measured by PIT from 0 to 0.12 shear strain γ.

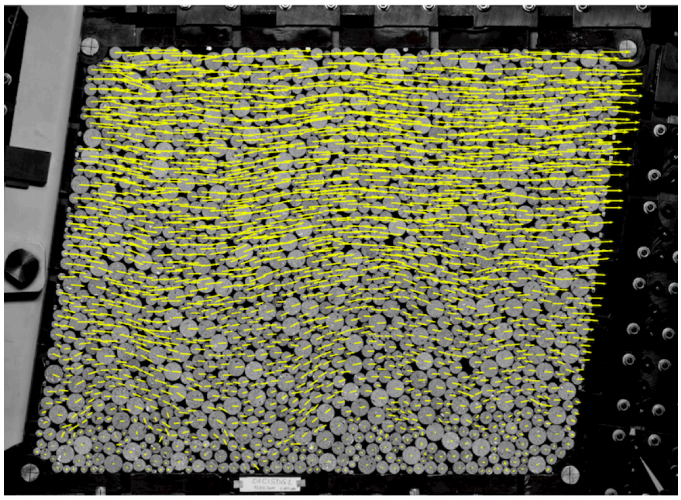

Figure 2: Particles displacement (maximum displacement = $46mm$) in a sheared 2D granular assembly made of 2000 wooden rods. The granular packing is enclosed by a rigid frame initially rectangular ($0.56m \times 0.47m$). A speckle of black and white points is painted on each cylinder to allow the measurement of particle kinematics by means of the PIT technique.

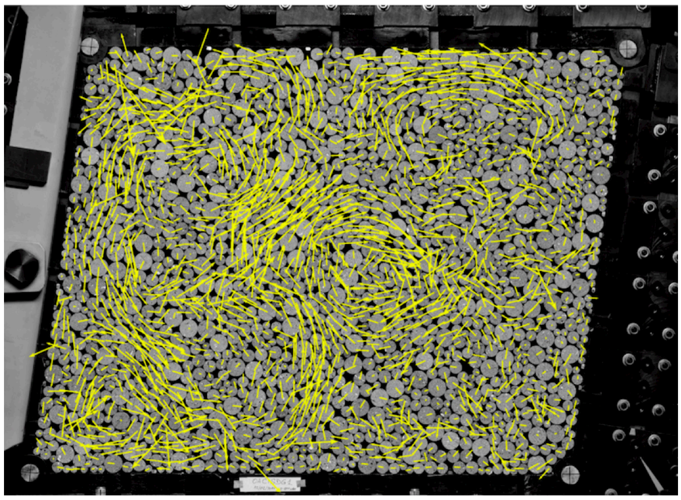

Figure 3: Displacement fluctuations $u(\Delta\gamma = 0.1, \gamma = 0)$ of grains in a sheared 2D granular assembly. The corresponding grains' displacement are shown in Fig. 2. The maximum value of u is $18mm$.

3 FLUCTUATIONS OF DISPLACEMENT

To assess the fluctuations of displacement (angular rotations of grains are also assessed, but not discussed in this paper) in the course of deformation, we consider two possible displacements of each grain during a strain window $\Delta\gamma$ (always positive). The first is the actual displacement vector $\delta\mathbf{r}(\gamma, \Delta\gamma)$, which depends both on the size of the strain window $\Delta\gamma$ and the level of shear strain γ at the beginning of the strain window. The second displacement vector $\delta\mathbf{r}^*(\gamma, \Delta\gamma)$ is the displacement dictated by a homogeneous (*affine*) continuum strain field, *i.e.*, the displacement that the grain's center would have if it moved as a material point in a continuum. The fluctuation of the displacement is defined as the difference between these two displacement vector:

$$\mathbf{u}(\gamma, \Delta\gamma) = \delta\mathbf{r}(\gamma, \Delta\gamma) - \delta\mathbf{r}^*(\gamma, \Delta\gamma). \quad (1)$$

Figure 3 shows a map of the fluctuations u associated with the displacement field of Fig. 2. Displacement fluctuations can be conveniently normalized by dividing the vector $\mathbf{u}(\gamma, \Delta\gamma)$ by the product $\Delta\gamma \langle d \rangle$ (where $\langle d \rangle$ is the mean diameter of the grains), which can be interpreted as the average displacement of the grains in the strain window $\Delta\gamma$. This normalized fluctuation,

$$\mathbf{V}(\gamma, \Delta\gamma) = \frac{\mathbf{u}(\gamma, \Delta\gamma)}{\Delta\gamma \langle d \rangle}, \quad (2)$$

can also be interpreted as a local (microscopic) strain fluctuation, which is in turn divided by the size of the global (macroscopic) strain window $\Delta\gamma$.
The mean displacement fluctuation $\langle u \rangle$ increases monotonically throughout the test, from 0.066 mm (for $\Delta\gamma = 10^{-3}$) to 6 mm (for $\Delta\gamma = 0.25$), this final value correspond to about 10% of the average displacement of the grains between the beginning and the end of the shear test. Note that the smallest displacement fluctuation is well above the accuracy of TRACKER. As far as the mean magnitude of normalized fluctuations $\langle V \rangle$ is concerned, a decrease of $\langle V \rangle$ from 3 to 2 is observed; this implies that the average fluctuation of local shear strain $\langle u \rangle / \langle d \rangle$ is two to three times larger than the global strain window $\Delta\gamma$.

4 PDF OF THE FLUCTUATIONS

Figure 4 shows the probability density function (pdf) of normalized fluctuations $\mathbf{V}$ along the x direction for two different strain windows $\Delta\gamma$. Whatever the strain window, a good candidate for the *pdf* is

$$F(V_x, q) = a\left[1 + b(1-q)V_x^2\right]^{\frac{1}{1-q}} \quad (3)$$

This function, named *q-gaussian*, comes from the generalized thermodynamics theory, which is derived from the principle of non-extensive entropy introduced by Tsallis 1988. The parameter q in Eq. 3 quantifies the degree of non-extensivity of the entropy of the system. In the limit of $q \to 1$, the function tends to a *Gaussian* probability density function.
Statistical analysis of the fluctuations shows that whatever the value of $\Delta\gamma$, their spatial average is zero, which is expected for a global homogeneous deformation. A key observation is that the pdf exhibits a wider range of fluctuations with decreasing $\Delta\gamma$, which corresponds to a variation of q from 1.14 to 1.59. In Richefeu et al. 2012, a *Kurtosis* analysis of V_x values was seen to tend to zero asymptotically with increasing size of $\Delta\gamma$. A zero value of kurtosis might indicate that the pdf tends to become Gaussian for $\Delta\gamma \to \infty$, which is confirmed by the decrease of q when $\Delta\gamma$ increases. Very similar *Kurtosis* features and trends have been observed (on DEM simulations) by Radjai and Roux 2002, who noted that they have striking similarities with turbulent flow of fluids (although no dynamics are associated with these fluctuations).

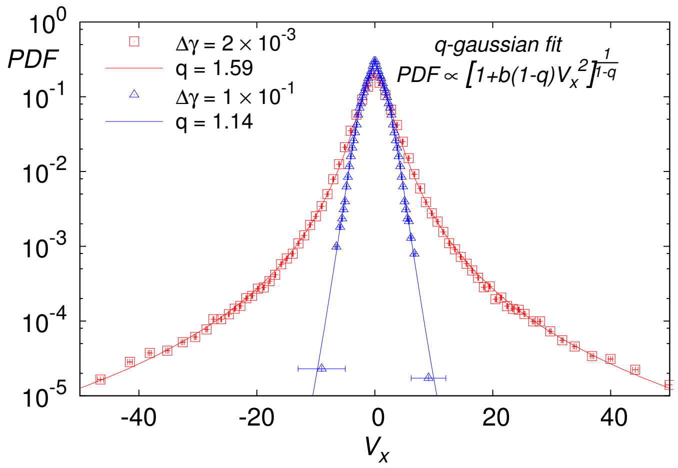

Figure 4: Probability density function (pdf) of normalized fluctuations V_x projected on the horizontal direction for two different strain windows, $\Delta\gamma = 2 \times 10^{-3}$ and $\Delta\gamma = 10^{-1}$.

5 SPATIAL CORRELATIONS OF THE FLUCTUATIONS

To understand the fluctuations, the analysis of their statistics must be supplemented by the study of their spatial distribution.

Spatial correlation of $\mathbf{V}$ can be seen, *e.g.*, the "loop patterns" in Fig. 3. Figure 5 shows three normalized fluctuation maps measured for three different values of γ, for two different sizes of the strain window $\Delta\gamma$. On the left, the two fluctuation maps computed for $\Delta\gamma = 2 \times 10^{-3}$ at two different values of γ show that both long–range correlation (blue curve, 40 grain diameter of correlation length) and short–range correlation (red curve, 10 diameter of correlation length) are both observed. This is consistent with *Tsallis* theory, which states that when q-Gaussian distributions are observed with $q > 1$ (here $q = 1.59$), then long-range spatial correlations emerge. On the right of Fig. 5, the strain windows used to compute the normalized fluctuation is bigger ($\Delta\gamma = 1.7 \times 10^{-1}$); the statistical distribution of V_x is similar to the one shown in Fig. 4 for $\Delta\gamma = 10^{-1}$. The q value of the q-Gaussian fit is smaller than for $\Delta\gamma = 2 \times 10^{-3}$ ($q = 1.14$), but still greater than 1 (perfect *Gaussian* distribution). A strong spatial correlation of V_x is found for distances less than $\Delta = 10$ particle diameters. This length roughly corresponds to the minimum radius of the fluctuation loops. A pseudo-period of about $\Delta = 20$ diameters is observed, which is another signature of the loops. This pseudo-periodicity might indicate the existence of a cascade of loop-sizes rather than a unique size – the smallest size being about 10 diameters, and the largest in the order of the sample size. This interpretation was confirmed by Richefeu et al. 2012 with a Fourier transform of the fluctuation signal as a function of space, as originally suggested by Radjai and Roux 2002.

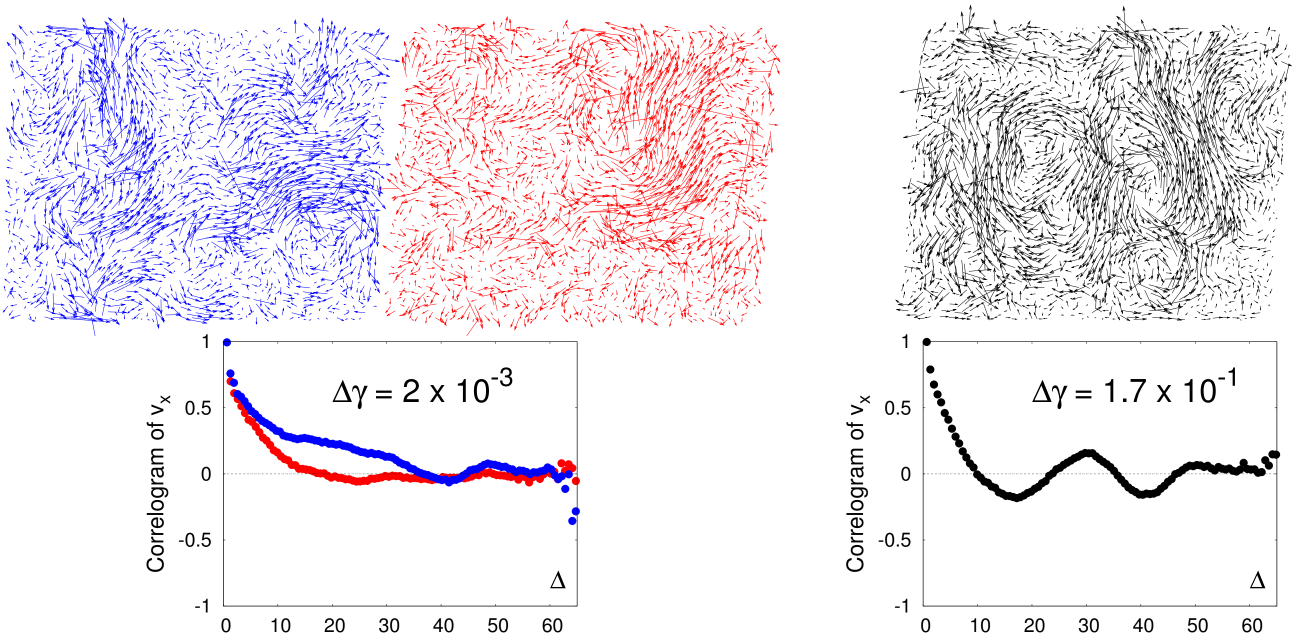

Figure 5: Maps of the normalised fluctuations for two differents values of $\Delta\gamma$. The curves show the spatial auto-correlogram for the x component of **V**. Δ is the correlation length expressed in terms of grain diameters.

6 CONCLUSIONS

A detailed study of fluctuations of displacements in a granular material subjected to quasistatic shear has revealed that the fluctuations are organized in space and show clear vortex-like patterns, reminiscent of turbulence in fluid dynamics. While these fluctuation loops have been often observed in DEM numerical simulations, on the experimental front they have only been reported (to the authors' best knowledge) in the study by Misra and Jiang 1997. The present study (results of which have already been presented in Richefeu et al. 2012) confirms these early findings and brings into the picture the important idea that fluctuation loops have a characteristic minimum radius – equal to 10 mean particle diameters for the material tested. Displacement fluctuations in granular materials are a direct manifestation of grain rearrangement, therefore they can be thought of as the basic mechanism of irreversible deformation. The statistical analysis of these fluctuation was described by q-Gaussian probability distribution, which is a signature of long-range correlation for $q > 1$. The link between these fluctuations (and their spatial organization) and the deformation of granular materials at the macro scale will be investigated in futur work.

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