An Attempt in Assessing Contact Forces From a Kinematic Field

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Abstract. In granular materials, it is not so simple to assess experimentally the contact forces. Photoelasticity is generally used for this purpose but this technique involves some constraints that may limit its use. We propose a different solution, which implements both the digital image correlation (DIC) technique and the non-smooth contact dynamics (NSCD) formalism. In a nutshell, the technique aims to find a set of contact forces mechanically admissible given a set of measured contact velocities. We used photographs of a simple shear test of a two-dimensional analogue granular material (about 1000 aluminum rods) to apply the solution, and we showed that valuable information about the contact forces can be extracted from the kinematic field provided that no major rearrangement occurs for at least five image shots.

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INTRODUCTION

The very peculiar nature of force transmission in granular materials is well known today. The study and the experimental measurement of these forces are thus of primary importance. In practice, there is few research team that are committed to this task. On the other side, the study of the grain-scale kinematics of these materials is most widespread. Particularly, the device $1\gamma 2\varepsilon$ which is basically a two-dimensional parallelepiped that allows for any quasistatic straining on Schneebeli analogue materials, was used to study, for instance, the structure evolution under complex loading path [1, 2].

The experimental method most commonly used to determine the contact forces is photoelasticity [3]. Unfortunately, this method cannot be applied with the $1\gamma 2\varepsilon$ device which makes use of 6 cm long cylindrical rods. The picture (tension lines) of stress distribution within the rods would be inaccurate due to possible misalignment of the quite long contact lines.

A much less common solution was initiated by Andrade and Avila [4]: the *granular element method* GEM. It is an analytical method which is able to determine the contact force acting on a volume (a rod surface) from the knowledge of the stress field and one force at least on its boundary. Assuming a constitutive model, the stress field can be determined from strain field which is itself determined from DIC displacements. Unfortunately, with this approach, the number of rods in the system is limited since each rod surface has to be sufficiently discretized, and the measured displacement field is a noisy signal that need to be smoothed (not obvious when dealing with abrupt variations due to the confinement of strains at the vicinity of contact points).

We thought about yet another solution based on two techniques: the *digital image correlation* (DIC) technique to determine the relative velocities at contact levels, and the *non-smooth contact dynamics* (NSCD) technique to find a set of contact forces both compatible with the rod kinematics and the overall loading. Despite a possible non unicity of solution inherent to the NSCD, we expect that, even if several sets of contact forces are compatibles, they all share the same features – at least from a statistical point of view. The combined use of displacement measures with an implicit resolution of forces was conceived thanks to the recent development of an accurate DIC technique – so called *particle image tracking* (PIT) – adapted to the non-smooth kinematics involved of a collection of grains, which is detailed in another paper of the present proceeding (Powders & Grains 2013).

BASICS OF NSCD

The NSCD method is described in depth in the literature, e.g. [5, 6, 7, 8]. As explained in [8], an implicit time-stepping scheme can be derived from a consistent description of the dynamics at the velocity level. We will however not enter into details here, but rather highlight the key aspects involved in this approach, in the particular context of its use as an indirect method of force determination with experimental data.

Within the scope of the NSCD, the time interval δt represents a unit of time during which velocity jumps can occur. The challenge of the problem is to predict, from a given configuration (connectivity and full kinematics of grains), the contact forces $\vec{f} = f_n\vec{n} + f_t\vec{t}$ over the elapsed time between each photographs. It is important to stress that contact forces used in the following correspond to *filtered* values of force impulses – that expresses as the product of force by time – during a time interval δt. It is obviously not possible to capture the force variations during the elapsed time δt between two consecutive acquired photographs (typically, a few tens of seconds); these periods are thus "blinded periods". As a consequence, since the forces assessed are associated with δt, and thus on the continuous-shooting speed, their informative content is poor in the cases of non-persistent contacts corresponding to successive collisions or rearrangement events. Fortunately, long-lasting contacts should withstand quasistatic forces that are not very sensitive to δt.

As the forces, the relative velocity between grains also varies within the blinded periods. This lack of information has been accounted for in the early development of the NSCD by introducing the *formal velocity*. It corresponds to weighted average between two moments: $v_k = \eta v_k^- + (1-\eta)v_k^+$, where subscript k stands for normal (n) or tangential (t) direction, η is the weight factor, v_k^- and v_k^+ correspond to the relative velocities at the beginning and the end of a time increment. If we set the restitution coefficient regardless of the direction k as $e_k = -v_k^+/v_k^-$ for binary contact, η reads: $\eta = e_k/(1+e_k)$. A restitution coefficients is physically limited in the range [0 1], which makes the weight factor η defined in the range [0 0.5].

The equation of dynamics is formulated by a single contact equation – in which velocity jumps replace the accelerations – at the contact level. It formally reads:

$$\begin{pmatrix} f_n \\ f_t \end{pmatrix} = \mathbb{W}^{-1} \begin{pmatrix} v_n/[(1-\eta)\delta t] + a_n \\ v_t/[(1-\eta)\delta t] + a_t \end{pmatrix} \tag{1}$$

where $\mathbb{W}$ is a matrix of inverse reduced inertias that depends only on the contact geometry (i.e. contact position and normal orientation) and inertia parameters of the involved grains. The values of a_n and a_t depend on the left-limit velocities v_k^- and the surrounding forces, i.e. the other forces acting on the grains implied in the contact; Figure 1.

In addition, the method implies in its generic formulation two basic kinematic constraints when the discrete elements are in contact:

1. The Signorini conditions (velocity version) stating that the normal force f_n is *repulsive* or *null* when the relative normal velocity v_n between two grains is zero. Otherwise, $f_n = 0$.
2. The static friction implying a coefficient μ limiting the friction force f_t by $\pm\mu f_n$ independently of the sliding velocity v_t (excepted its direction). In case of non-sliding contact, f_t can take *any* value in the interval $[-\mu f_n\ ;\ \mu f_n]$.

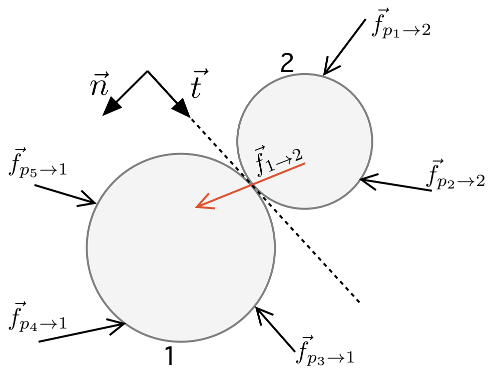

FIGURE 1. Two grains in contact and its "surrounding contacts". The method consists basically in finding, over all the contacts of a packing, the forces that obey both the dynamical equilibrium of single contact configurations and the kinematic constraints.

At each time step, all velocities and contact forces must be resolved by taking into account, simultaneously, all kinematic constraints implied by enduring contacts. It practically consists in looping over the contacts, computing the contact forces by setting a formal velocity equal to zero in Equation (1), and replacing inconsistent force values by a value that obey the kinematic constraints:

$$\begin{array}{rcl} f_n < 0 & \mapsto & f_n = 0 \\ f_t < -\mu f_n & \mapsto & f_t = -\mu f_n \\ f_t > \mu f_n & \mapsto & f_t = \mu f_n \end{array} \tag{2}$$

For a given contact, the *single contact problem* is the consideration of both the dynamic (Equation 1) and kinematic constraints (Equation 2). We use the nonlinear Gauss-Seidel method to solve each single contact problem, with other contact forces being treated as known (see Figure 1), and iteratively updating the forces until a given convergence criterion is fulfilled. The iterations are stopped when the forces are stabilized, that is, when the force variation between successive iterations divided by the current force is within a prescribed precision criterion ε_f for *each* contact.

FEASIBILITY STUDY

Reliability of NSCD was already demonstrated in the past. One can easily predict that the application of the method with experimental data will give result anyway. As a consequence, to use the NSCD in the inverse method proposed herein, the first relevant question is how the set of forces is affected by inaccuracies inherent in experimental measurements and very large lag-times in the continuous-shooting procedure.

Experiments and data extraction

The two-dimensional (2D) equivalent media used within the $1\gamma2\varepsilon$ apparatus was made of 6 cm long aluminum rods; Figure 2. We performed a simple shear test with a constant vertical pressure $\sigma_y = 50$ kPa, corresponding to a single loading cycle with an amplitude of 15 deg. at constant shear rate $\gamma = \pm 4.8 \times 10^{-3}$ deg. s^{-1}. This shear rate was chosen so that the loading is quasistatic according to the inertial number criterion [9].

In order to identify the centers of "circular grains" and track them, a pattern was painted on the front side of each rod with black speckle onto a white background (see inset in Figure 2). The center position together with the solid rotation of each grain are followed throughout the test by means of the PIT technique [10]. The positions are thus assessed with a precision of about 10 μm. To identify the positions of the contact points and the corresponding normal directions, we used a simple geometric rule that involves the center positions and radii of each rod pairs. A slight variation in all radii was used as control parameter to tune the connectivity (and its evolution) according to familiar and specific features of granular packings (in particular, coordination number close to 4). When applying the NSCD, the force solution should be particularly sensitive to the properness of the contact point identification. We paid therefore a particular attention to this automatic procedure by comparing its result with a solution determined manually for a few images.

Anyway, the sources of errors are twofolds: (1) inaccuracies in input data, (2) wrong choice of contact parameters. To overcome these sources of errors, a sensitivity study is required. We do not present this study in this short communication but rather a first outcome that shows the applicability of the method. According to this goal, we did not focus, at first, on the contact parameters (η and μ). The parameter η was set to zero since most contacts persist. The friction coefficient μ was roughly estimated to 0.25 by performing some experiments.

A first outcome

The method was tested with two different boundary conditions : (1) measured velocity imposed for all boundaries, (2) constant pressure σ imposed on the top and measured velocity imposed elsewhere. The data used with the first condition come directly from PIT measures which are quite accurate (if compared with those from sensors), the drawback being a loss in force-scale. On the other side, the second condition should allow the contact forces to scale with the force imposed on the top plate. We checked that the solutions obtained with both boundary conditions were similar (in the "uniform scaling" sense). Since the force sensor were initialized with the sample in place, we did not account for the gravity.

Figures 3(a) and 3(b) show different patterns of the contact force network obtained after the convergence criterion is fulfilled, the initial forces being set to zero. There exists a patent distinction between solutions that look plausible and pretty consistent and those that certainly are not. To a large extent, the latter must be due to the vagueness of the experimental inputs. Even for acceptable solutions – which are fewer – nothing guarantees they are the appropriate ones.

With the NSCD, although uniqueness of the solution at each time step is not guaranteed, initializing each step with the forces calculated in the preceding step makes the variability of admissible solutions shrink to the numerical resolution. We decided to use the same strategy to overcome the problem of inconsistent solutions. The results showed the expected behavior after about 5 images (or steps) as shown in Figure 4: most patterns of force network seem consistent and evolve slowly from one image to another, excepted for some particular moments where sudden (cooperative) rearrangements take place. In these cases, the initial guess must be reset to the zero-forces configuration, and the 5 following images are still required to obtain consistent solutions. To summarize, the contact forces are in principle extractable from a series of consecutive images corresponding to a period without sudden rearrangements. From a quantitative viewpoint, it is interesting to notice that macroscopic friction angles obtained from both experimentally recorded and identified forces, were about the same. Similarly, satisfactory agreements were obtained for the principal directions of the stress tensor, and the distribution of normal forces was consistent: large extent of the forces (from 0 to 5 times the mean value), exponential tail, and a plateau of weak forces that reflects the anisotropy of the packing mainly due to the nearly monosized distribution of grains [11].

CONCLUDING REMARKS

We presented a novel technique to assess the contact forces involved in a slowly strained granular packing. This technique, which is at an early stage of development, seems promising. It is worth noting that a quantitative estimate of the static forces is possible although multiple photographs of close but different configurations are required. However, since the method combines experimental data with a numerical resolution, an outstanding issue will be to test its robustness with respect to the input data that pertain unavoidable inaccuracies. A sensitivity analysis needs to be conducted to address this question. We also plan to test the method on synthetic data that will be generated by means of discrete element

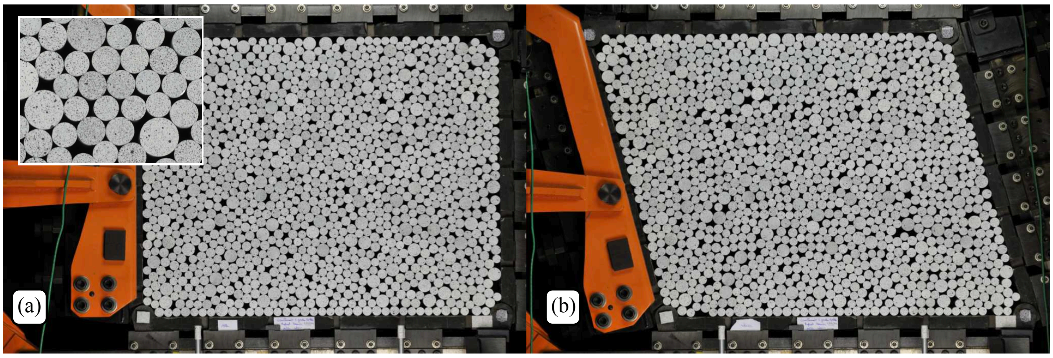

FIGURE 2. The apparatus $1\gamma2\varepsilon$ in two different configurations during a simple shear test: (a) $\gamma = 0$ degree; (b) $\gamma = 15$ degree. The sample is a 2D granular assembly of 1172 aluminum rods (diameters equal to 14, 16 and 22 mm), enclosed by a rigid frame initially rectangular (0.5 m $\times$ 0.6 m). Inset: a view of the speckle pattern painted on the rod sides, required to track their positions and rotations.

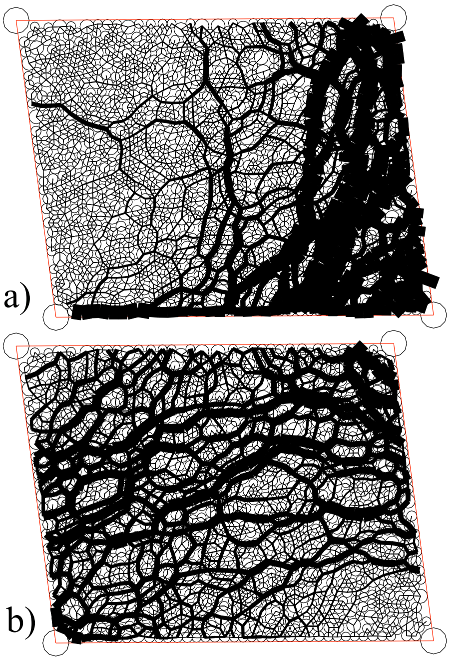

FIGURE 3. Typical maps of contact forces initially set to zero at the beginning of the iterations: (a) a pathological case mainly due to inaccurate inputs; (b) a solution that seems more consistent.

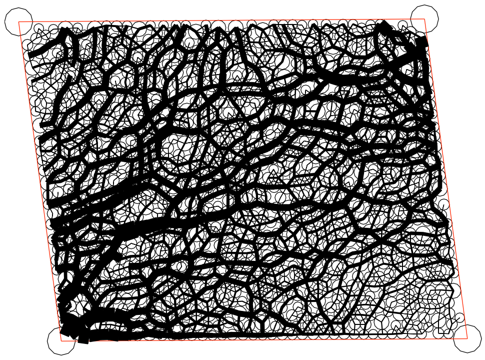

FIGURE 4. Typical maps of contact forces initialized with previously found forces at the beginning of the iterations.

simulations based on an explicit integration scheme.

REFERENCES

1. F. Calvetti, G. Combe and J. Lanier, *Mechanics of cohesive-frictional materials* **2**, 121–163 (1997).
2. E.-M. Charalampidou, G. Combe, G. Viggiani, and J. Lanier (2009) in Powders and Grains 2009, Ed. by M. Nakagawa and S. Luding, Golden, USA, pp. 821–824.
3. T. S. Majmudar and R. P. Behringer, *Nature (London)* **435**, 1079 (2005).
4. J. E. Andrade and C. F. Avila, *Granular Matter* **14**, 51–61 (2012).
5. M. Jean, J. J. Moreau (1992). “Unilaterality and dry friction in the dynamics of rigid body collections.” in: Proceedings of Contact Mechanics International Symposium, Lausanne, Switzerland, pp. 31–48.
6. J. J. Moreau (1993). “New computation methods in granular dynamics,” in *Powders & Grains 93*, Rotterdam, A.A. Balkema, p. 227.
7. M. Jean (1999). “The non smooth contact dynamics method,” in *Special issue on modeling contact and friction.*
8. F. Radjai, and V. Richefeu, *Mechanics of Materials* **41**, 715–728 (2009).
9. G. Combe and J.-N. Roux (2003). “Discrete numerical simulation, quasistatic deformation and the origins of strain in granular materials,” 3rd International Symposium on Deformation Characteristics of Geomaterials, 2003, Lyon, 1071–1078.
10. V. Richefeu, G. Combe and G. Viggiani, *Géotechnique Letters* **2**, pp. 113–118 (2012).
11. V. Richefeu, E. Azéma, F. Radjai, M. S. El Youssoufi, *Powder Technology* **190**, 258–263 (2009).