International Workshop
HOMOGENIZATION, THEORY OF MIGRATION AND GRANULAR BODIES
Gdańsk - KORMORAN, 14-17 May 1995

An experimental study of deformation in 2D granular media.

J.Lanier, G.Combe
Laboratoire 3S, Grenoble, France.
Reseau de laboratoires GEO.

Abstract :

In recent past years, micromechanics of granular media was a subject of increasing interest. The question is :" Is it possible to define the macroscopic behavior of such material from basic mechanics of grains ?". Many results are obtained from theoretical and numerical points of view, but very few real experiments were performed. In this paper the deformation of an assembly of 300 rods is described. The macroscopic deformation on the boundary (compression or simple shear) is imposed with a special shear apparatus. Rotation and displacement of each rod are measured. This experimental procedure allow an analysis of local deformation of the assembly (geometry and evolution of contacts, rolling and sliding, localization of deformation) in relation with kinematics of the material considered as a continuum.

Introduction :

Granular media are an assembly of grains which are often considered as rigid. At a large scale (macro scale), the description of such media is performed in the framework of continuum mechanics. Constitutive equations which are obtained from experimental results are phenomenological and internal variables (plastic strain, hardening parameters..) are introduced to obtain a realistic modelling (elastoplasticity). The physical meaning of such variables is generally not clearly explained.

On the contrary, at the micro scale, the local variables are very well identified : the structure of the assembly is caracterized by the shape of the grains, the number and the orientation of contacts (geometry of the set).The kinematics is completly defined by the displacement and the rotation of grains (considered as rigid bodies) and the rheological properties are the contact law (essentially friction and unilateral contact).

The definition of continuum state from local variables is a difficult problem which is generally solved in the framework of homogenization or statistitics approaches (Christoffersen et al.-1981, Cambou -1993, Oda -1982, Caillerie - 1991, 1993)

On the other hand, many interesting results were obtained numerically with Distinct Elements Method (Cundall et al.- 1982; Rothenburg and Bathurst -1989, 1992; Thornton - 1990, Jean and Yemmas - 1992; Cambou and al.-1994; Bardet, Proubet - 1991).

This paper is concerned with experimental results about the deformation of granular assembly. Macroscopic boundary conditions are imposed on a specimen of rods (Schneebelli material) which allow the definition of global stress and strain. At micro scale, displacement and rotation of 300 rods are measured and an analysis of mecanism of deformation is performed, especially : evolution of contacts number and their orientations according to the direction of principal stresses, rolling and sliding, correlation with the macroscopic strain and size of elementary representative volume for the use of continuum mechanics, strain localization at microscale..

Test procedure :

The specimen is a 2D-material of wood rods. Three diameters were used : Φ = 13, 18, 28 mm. This specimen is deformed on a special shear apparatus (see fig.1) (Joer and al.-1992) which allows general conditions of plane strain and measurements of boundary conditions in terms of macro-strain (ε_x, ε_y, ε_{xy}) and stress (σ_x, σ_y, σ_{xy}). From the micromechanics point of view, 300 rods (100 of each diameter) were observed. Photographs are taken at different steps of the deformation. On each photograph the position of each rod is measured. So, the number and the orientation of contact, the displacements and rotations may be deduced.

Two kind of tests are performed : compression with constant lateral pressure, and simple shear (σ_n = constant).

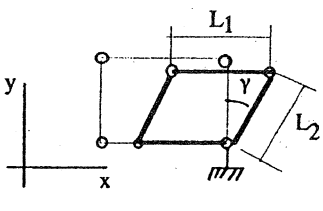
Experimental results :

1 - Evolution of the number of contacts and their orientation :

All along the deformation of the specimen, the mean coordination number <N> remains close to 4 with a tendancy to decrease, because of the global dilatancy of the specimen. A more complete analysis may be performed : for each grain, during the deformation, contacts may be maintained, lost or gained. Figure 2 gives the histograms of gained (g) and lost (p) conctacts versus the contact-orientation for a simple shear test (γ = 22 %). A second order Fourier approximation is plotted on the same figure :

$N(x) = N_o.(1 + d.\cos(2(x-x_o)))$ with : $-90° < x < +90°$ and $d > 0$

We conclude that the maximum of contacts are lost in the direction x= -48° and gained in the direction x = 45°. These 2 directions are close to the principal directions of extension and compression predicted by continum mechanics. This evolution of the contact orientation induced an evolution of the geometrical anisotropy of the structure (fabric tensor for example). It is interesting to notice that the two curves intersect for x = 0° and x = 80°. According to continuum mechanics these two directions are very close to zero-extension lines.



550 < L1 < 687 mm
413 < L2 < 550 mm
-20° < γ < + 20°

Figure 1 : Principle of the shear apparatus.

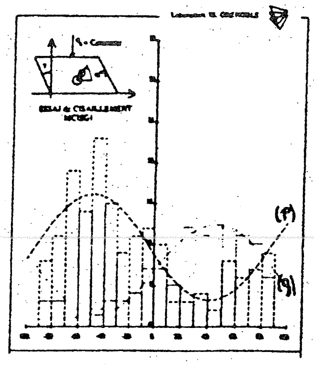


Figure 2 : Histograms of lost and gained contacts. (test MCISG1)

2 - Sliding and rolling at the contact of two rods

Let us consider 2 rods which remain in contact during the deformation (fig.3). We define the two parameters : $a = CC_1$ and $b = CC_2$ where C is the contact point in the deformed configuration and C1, C2 the material points of rods 1 and 2 which were in contact in the initial configuration. The rolling without sliding condition is expressed by :

$$a + b = 0$$

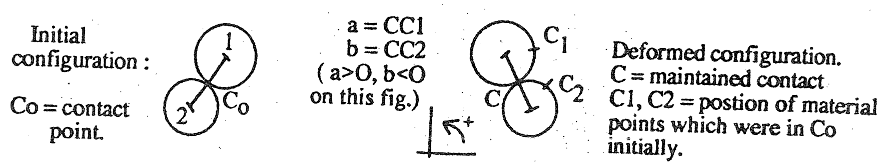


Figure 3 : Definition of a and b; (a+b) = 0 is the rolling without sliding condition.

The figure 4 is the plane (a,b) for all the contacts in the assembly (same shear test as previously). In this figure each contact point gives 2 symetrical points (if rod i is in contact with rod j, then rod j is in contact with rod i !!). On this figure is is clear that the main part of contacts which are maintained during the deformation are close to rolling without sliding condition.

On figure 5 are plotted the links between rods which are such that the sliding (a+b), at their contact point, is less than 10% of $(a+b)_{max}$. This figure is very similar to that of forces lines which are observed by photoelastic method (direction of major principal stress). We can conclude that the rolling without sliding condition occur on contact which are oriented close to the major principal stress (in compression)

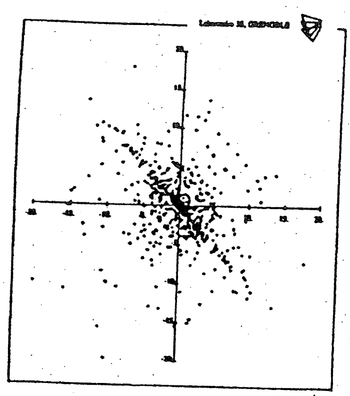


Figure 4 : Plane (a,b). The rolling without sliding condition is a+b = 0

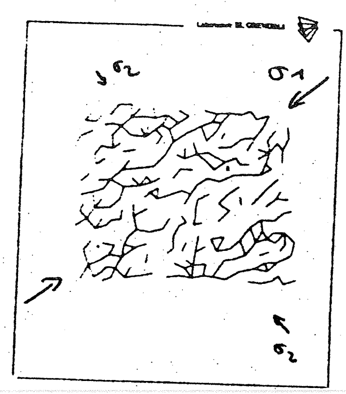


Figure 5 : Links between rods whose contact verify the rolling without sliding condition (simple shear test)

3 - Experimental definition of an elementary representative volume :

According to continuum mechanics the mean deformation gradient $< u_{i,j} >$ inside a domain (D) may be define by the value of the displacement field u_i on the boundary by :

$$< u_{i,j} > = \frac{1}{S}\int_{(L)} u_i . n_j . dL$$

where (L) is the boundary line, S is the surface of the domain (D) and n_j is the exterior normal along the line (L).

Our measurements give the displacement field of each rod center . So we can apply the previous formula by choosing as line (L) a polygonal line defined by centers of rods. The displacement field along this line is known at each summit (centers of rods), and is interpolated linearly along the side. For each of the 300 rods, we define a family of polygonal lines (L) with increasing size, around the rod under consideration.

The calculus of $<u_{i,j}>$ for each line (L) give the macroscopic strain (ε_x, ε_y, ε_{xy}) and the macroscopic rotation ω :

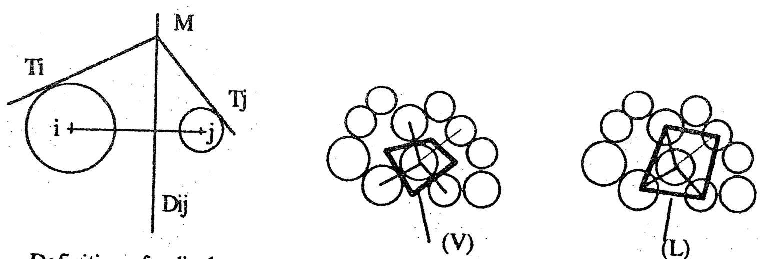
$$\varepsilon_{ij} = 1/2.(<u_{i,j}> + <u_{j,i}>) \quad \text{and} \quad \omega_{ij} = 1/2.(<u_{i,j}> - <u_{j,i}>)$$

It can be notice that this approach does not take into account the rotation of grains

On figure 6 these local strains and rotation are plotted versus the surface S of the domain (D).The same tendancy are observed for each plot : for small domain (D) the calculated values present a large scatter which decreases when the size of domain (D) increases. The limit value is close to the macroscopic value imposed by the boundary conditions. When the scatter is small enough, the size of the domain (D) may be consider as a elementary representative volume for continuum mechanics. In our case this size seems to be 10 to 15 grains.

4- Localization of deformation at microscale :

For grain of the assembly, we define a 2D-Voronoï tessallation : the Voronoï cell of grain n° i is obtained by consideration of radical axes of grains i and j (j = 1,N). Among these axes we select those who defined the convex polygon line closest to grain n°i. From the cell, grains contiguous to grain n°i are identified and allow the definition of a polygon line (L) around grain n°i (figure 7). This line (L) is used to calculate (as in § 3) the local deformation (ε_x, ε_y, ε_{xy}) and the local shear intensity $E = (\varepsilon_1 - \varepsilon_2)$ where $\varepsilon_1 > \varepsilon_2$ are the principal values of local deformation.



Definition of radical axe Dij for Voronoï cell : Dij = (M / MTi = MTj)

Voronoï cell (V) and polygon line (L) defined with contiguous grains for calculus of local deformation.

Figure 7 : Definition of : radical axe for two circles, Voronoï cell for grain n°i and polygonal line (L) of contiguous grains around grain n°i.

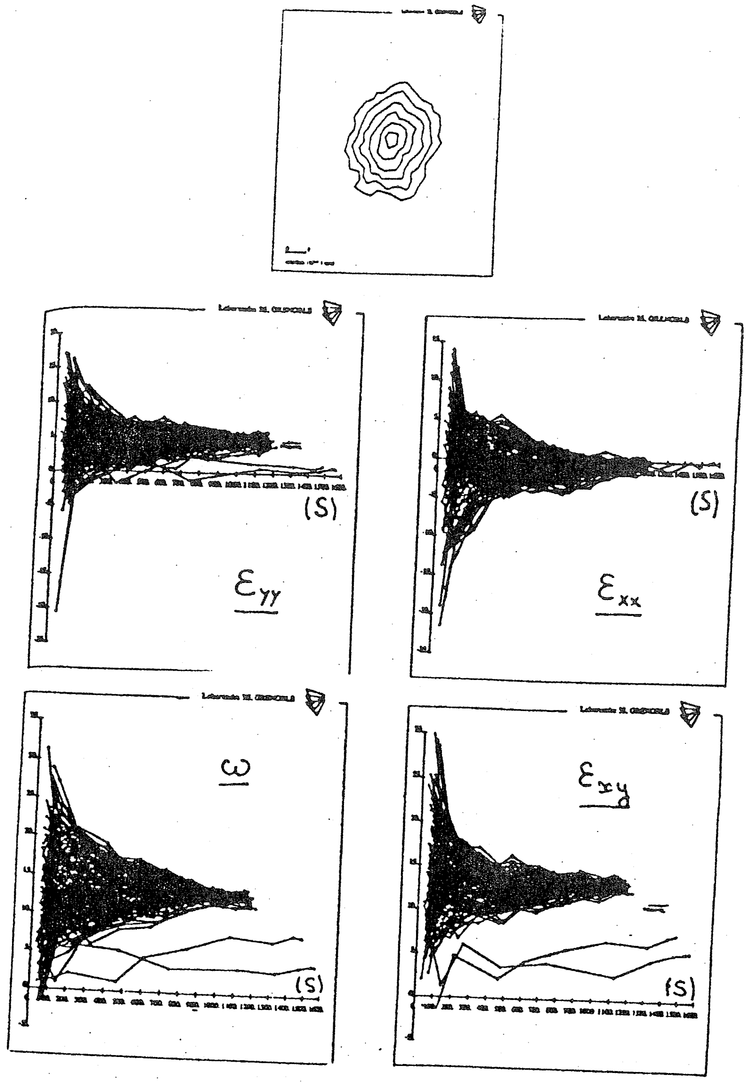
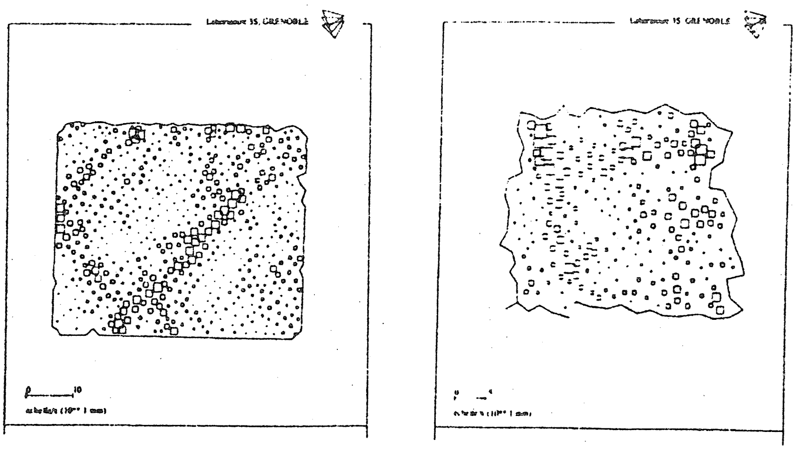


Figure 6 : Definition of an elementary representative volume : evolution of strain and rotation versus the size of domain (D) - (test MCISD2, 1-5)

A map of local shear intensity E(x,y) is plotted on figure 8. For each grain the local value of E is indicated by a square whose size is proportionnal to E-value. This map shows local heterogeneity of deformation and it is clear that this heterogeneity is organized in shear band. The thickness is not more than 5 grains and, according to the previous result of §3 continuum mechanics cannot be applied inside the shear band : its size is less than the size of the elementary representative volume.



(a) vertical compression (GCV1 2-3 ; 722 rods)

(b) simple shear test (MCISD2 1-5 ; 300 rods)

Figure 8 : Maps of local shear intensity

Conclusion

Micromechanical experiments are very important to understand the macroscopical behavior of granular media. We have shown that induced anisotropy is well explained by the fact that in deviatoric loading many contacts are lost in the direction of extension, contacts are gained in direction of compression. So the structure becomes geometrically anisotropic.

Granular materials are discret materials by definition. The classical approach in the framework of continuum mechanics need a definition of an elementary representative volume. Our measurements indicate that the size of this volume is about 15 grains. It is important to notice that in the litterature, the proposed thickness of shear band in granular material is often of the same order. As a consequence, the study of shear banding by using continuum mechanics inside the band seems questionnable.

Références :

Bardet J.P, Proubet J. (1991) : A numerical investigation of the structure of persistent shear bands in granular media. Geotechnique, 41, N°4, 599-613.

Bonnet P.C (1993) : Etude micromécanique d'un assemblage de rouleaux, Mémoire de DEA, Université J.Fourier, Grenoble.

Caillerie D. (1991) : Tenseur des contraintes dans un milieu granulaire, Rapport d'activité Greco 1991, p 771,775.

Caillerie D.(1993) : Homogénéisation d'un milieu granulaire : loi incrémentale, GEO 94 (to be published)

Cambou B. (1993) : From global to local variables in granular materials. Proc.Powders & Grains 93 p73, 86. Thornton edt, Balkema

Christoffersen J., Mehrabadi M.M, Nemat-Nasser S. (1981) : A micromechanical description of granular material behavior. J.of applied Mechanics, Vol.48, p339-344.

Cundall P.A, Drescher A.,Strack O.D.L (1982) : Numerical experiments on granular assemblies; Measurements and observations, Proc.IUTAM conf.on Deformation and failure of granular materials p355, 370 - Vermeer, Luger edts, Balkema.

Jean M., Yemmas R. (1992) : Simulation numérique des milieux granulaires. Rapport d'activité GRECO 1992, p 743, 750.

Joer H., Lanier J., Desrues J., Flavigny E. (1992): "1g2e" a new shear apparatus to study the behavior of granular materials. Geotechnical Testing Journal, Vol.15,n°2, pp 129, 137.

Lanier J., Bonnet P.C (1993) : Experimental study of displacements and rotations of grains in a 2D-granular media. Meeting TC13, Comitee on mechanics of granular materials (ISSMFE), Birmingham.

Matsuoka H., Yamamoto S. (1993) : A microscopic study on shear mechanism of granular materials by DEM and experiments. Proc.Powders & Grains 93 p155, 160. Thornton edt, Balkema

Oda M., Konishi J., Nemat-Nasser S.(1982) : Experimental micromechanical evaluation of strenght of granular materials : effects of particle rolling. Mechanics of materials 1, p 269-283.

Reynaud M. (1994) : Micromécanique d'un assemblage de rouleaux, Mémoire de DEA, INPG, Grenoble.

Rothenburg L., Bathurst RJ (1989) : Analytical study of induced anisotropy in idealized granular materials. Geotechnique 39, n°4, 601-614.

Rothenburg L., Bathurst RJ (1992) : Micromechanical features of granular assemblies with planar elliptical particles. Geotechnique 42, n°1, 79-95.

Thornton C. (1990) : Induced anisotropy and energy dissipation in particulate material- Results from computer-simulated experiments. Yielding, Damage, and Failure of anisotropic solids, p113,130 - Boehler edt., Mech.Eng.Publications, London.