Pattern formation during capillary rising of a fluid front into a granular media

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Abstract. We report results concerning the pattern formation during the capillary rising of a fluid front into a dense dry granular media. The system consists in a modified Hele-Shaw cell filled with grains of different gradings and confined in a narrow gap between the glass plates. This assembly is vertically installed over a water reservoir to allow an ascending front of liquid to percolate into the granular media. We acquire digital images of the liquid/air front which are treated by means of imaging analysis techniques. Thus, we are able to assess the temporal evolution of the air/liquid boundary profiles. We measure the roughness of the profiles, using a detrended fluctuation analysis technique, to obtain their fractal dimension. We compare our results with similar experiments reported in literature considering fluid displacement into heterogeneous media and directed percolation theory. However, the range of values for the Hurst exponent obtained from our experiments are odd to the measured/predicted values in experiments or theory.

Keywords: capillary rising, granular systems, heterogeneous media, fluid displacement
PACS: 05.40.-a,81.05.Rm,64.60.ah

INTRODUCTION

Pattern formation is a key concept to understand how nature works, and the quest to access the basic knowledge of the underlying mechanisms of a wide range of phenomena has attracting the interest of researchers from different fields as biology, physics, engineering, computation etc. [1, 2, 3, 4]. In particular, the displacement of a fluid front into a heterogeneous media is an example of phenomena where the combination of theory, experiment and numerical modeling is able to successfully describe, qualitatively and quantitatively, the phenomena observed [5, 6, 7]. The fluid flow in heterogeneous media is an ubiquitous process and plays a central role on several natural phenomena and industrial processes as oil extraction, percolation, infiltration, chromatography etc. [8, 9]. An interesting feature is that the characteristic length and time scales related to phenomena can spans several orders of magnitude. The fluid flow on granular media is one of the most studied problems in this field last years, and a comprehensive theoretical description is arising [10, 11, 12]. However, an unified description for the large plethora of patterns observed still is lacking [13, 14, 15, 16].

Here, we report an experimental study of the air/liquid interface formed during the capillary rising of water into a granular, or porous, media. We present results for the fractal dimension of the interface and compare them to results obtained with previous experiments to study the air/liquid interface in heterogeneous media, and also with the predicted values of the Deppining by Directed Percolation model (DPD) [6, 4]. The assembly considered here is a modified version of a previous experimental setup where a fluid front percolates on the gap between two corrugated glass plates [17] – Figure 1. Basically, the setup used here consists of a Hele-Shaw cell filled with commercial sand of different gradings, placed over an water reservoir. We report some results for the Hurst exponent that are odd to literature. The paper is organized as follows: in the next section we present the Experimental Setup and Methodology. Section brings the results and the discussion and we finish with our conclusions and perspectives.

METHODOLOGY

The assembly considered is a modified Hele-Shaw cell made of two glass plates, with dimensions $(400 \times 300 \times 4)mm$, put apart by $4.0mm$ spacers placed at the four corners in the gap between the plates. Before to fill the cell with sand, the lateral borders were blocked with special bond tape, which also help to held the plates together. The narrow gap at bottom of the assembly was filled with a very fine polystyrene foam to avoid sand evasion to water. The sand used was a commercial washed

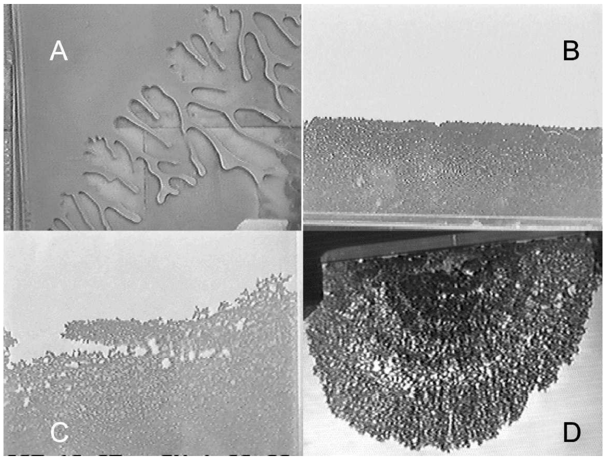

FIGURE 1. Fluid displacement in heterogeneous media. Examples of fluid displacement: (A) air displacing a moisture of glass, aluminum oxide and water; (B) and (C) water invading a narrow gap between corrugated plates at two different gradings - 80 *ppi* and 350*ppi* respectively; (D) India ink displacing air in a 500 *ppi* corrugated channel, towards the direction of gravity (downward).

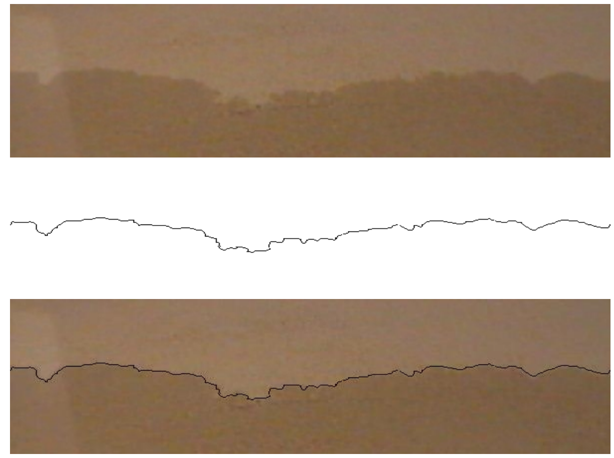

FIGURE 2. Image treatment of the air/liquid interface. The top panels shows the image captured from the digital camera, which was treated with ImageJ^{TM} in order to obtain a single valued function, shown in the middle panel. The bottom image combines the single value function and the original image to compare the efficient of the procedure.

sand, used in civil construction, with different gradings. Rough estimation of the grain sizes and geometries indicate that they are polyhedrons with average radii from few to thousand micrometers, high polydispersed, but a complete characterization of the granular media used is not available at this time (particle size distribution function). To fill up the cell, the sand was gently poured atop the cell, with help of a sheet of paper, and filled the gap slowly. The top side of the cell is left open and the sand was poured until it filled completely the gap. During the process, sometimes we tapped the system to compact and homogenize the media, but some degree of segregation is unavoidable. The granular media formed in the gap between the plates is very heterogeneous, and can be characterized as a highly connected porous media with strong capillary pressure.

The assembly was then fixed at the four corners with clamps. Then, we leveled the assembly to be aligned with gravity, and placed it over a plastic container partially filled with water. A video camcorder with 6.2 Megapixels was placed at 1.5m in front of the assembly and turned on just before the level of the water in the reservoir reached the bottom of the plates. The level of water was raised slowly and kept fixed when a flat interface was formed with the glass bottom. As a complete record of an experiment typically takes around 60 minutes, we disregard evaporation effects and consider the water-glass interface angle as constant - the volume of water of the reservoir (40L) is much bigger than the volume of water which percolates into the gap.

As soon the water touch the granular media, a liquid/air interfaced is formed. As result, there is a high contrast between dry and wet sand, making possible to identify the interface after some image treatment. The images were transfered to a PC using a 1394 connexion, and treated with ImageJ^{TM} software. Applying a sequence of numerical operations, as detailed in Figure 2, we extract a single valued function corresponding to the profile of the liquid/air interface, $h(x.t)$, where x indicates the horizontal position.

We have performed several runs considering exactly the same procedure described above, and took snapshots of the cell along all the experiment, at predetermined times distributed following a multiplicative scale. We analyzed each snapshot measuring its Hurst exponent H, using detrended fluctuation analysis (DFA) [18, 19], and taking the average of several snapshots to obtain $H(t)$. We consider the roughness as the fluctuation of the heights in the function $h(x,t)$ obtained after the image analysis,

$$w = \sqrt{\frac{1}{N}\sum_{i}^{N}\left(h_i - \overline{h}\right)^2} \quad (1)$$

where h_i corresponds to the height of the interface at the column i with pixel resolution. Considering now the roughness at local scale ε,

$$w(\varepsilon) = \sqrt{\frac{1}{1+2\varepsilon}\sum_{i-\varepsilon}^{i+\varepsilon}\left(h_i - (\alpha(\varepsilon)x_i + \beta(\varepsilon))\right)^2}, \quad (2)$$

where $\alpha(\varepsilon)$ and $\beta(\varepsilon)$ are the angular and linear coefficients which adjust the best linear fit at the local scale ε

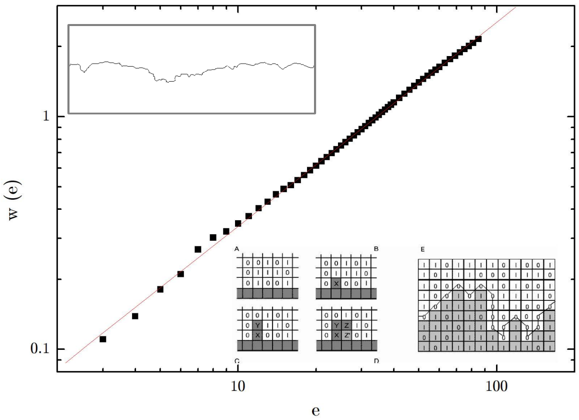

FIGURE 3. Hurst exponent calculation. The local width $w(\varepsilon)$ is plotted in function of the local scale ε. The continuous line indicates a power law fit giving the exponent $H = 0.917(1)$. The top right corner show the profile considered. The lower inset show the DPD model: the first successive steps of the model are shown at left, with the initial randomness of the material represented by 0 or 1 in each site, indicating that the water can flow or not by this site. After the first step, a site is filled and eventually wets all open neighbors. When a block site is placed under a wet site it became wet; this procedure avoid hangovers in the interface.

considered, it is possible to easily obtain the Hurst exponent from the relation,

$$w(\varepsilon) \sim \varepsilon^H \ , \tag{3}$$

as exemplified in Figure 3. Hence, applying this technique we are able to estimate the average Hurst exponent of the interface in function of time, as shown in Figure 4.

DISCUSSION

We shown in Figure 4 the summary of our results for the time evolution of the Hurst exponent of the air/liquid interface formed during the displacement of the fluids into a granular media, and compare with results obtained from the corrugated Hele-Shaw setup [17]. By direct comparison of the temporal evolution of the profile (interface velocity) we estimate that the granular media formed in the gap between the plates has a capillary force comparable to the experiments of corrugated plates obtained with 150 *ppi* aluminum oxide [17]. We arrived in this conclusion by direct comparison of the time scales on both experiments and looking for the closest match between the estimated interface velocities.

It is clear that the results from corrugated plates fluctuates much more than those from granular media. This can be understood considering the connectivity and dimensionality features in each case: the granular media is highly connected, and considerably wider than the corrugated gap. Thus, we can expect more 3D effects in the granular case, while the corrugated media is closer to 2D case. Besides, the capillary pressure on the corrugated case is highly dependent on the confining force of the walls, and it is practically impossible to repeat the exact experimental conditions. The granular case is much more reproducible than the corrugated case, since the gap between plates is keep constant, and the sand filling procedure leads to quite similar initial conditions in each run. Second, the high connectivity of the granular media allows the development of interfaces which are almost always single valued, in opposition to the corrugated case where, depending on the pressure on the plates, several hangovers and clustering patterns are observed spanning all the plate. These hangovers difficult the proper analysis of these interfaces, and leads to the large fluctuations in the measured Hurst exponents- Figure 1.b,c.

Another interesting observation is the odd values measured for the Hurst exponent in comparison to the observed in literature . While in the literature results span between $0.3 < H < 0.8$, in our experiments we consistently measured values of $H > 0.8$. We believe that, again, the high connectivity plays a central role, leading to smooth surfaces due to the 3D feature of the capillary media formed in the granular case. From the theoretic side, we believe that our system is well described by the DPD model, which is illustrated in Figure 3. The model prediction is $H \sim 0.633$, obtained from the ratio of the perpendicular and parallel correlation coefficients, $H \sim \frac{\nu_\parallel}{\nu_\perp}$, from the Directed Percolation universality class.

It is clear that our results can not fit with the model predictions. Maybe the difference between the morphologies of the grains and that of the pores (“voids”) in the experimental case could explain this disagreement. The granular media is formed by grains, which play the role of the blocked sites, and pores, or “voids”, between the grains, which can be described as the open sites of the model. Thus, we can expect a larger asymmetry in the experimental case than in the DPD model, since all sites have the same morphology. This extra asymmetry maybe can be reflected in the correlation length exponents.

CONCLUSIONS AND PERSPECTIVES

We report results for the temporal evolution of the Hurst exponent measured for the air/liquid interface formed during the capillary rising of water into a narrow granular media confined between two parallel glass plates. We compare the results with the predicted values of the DPD model, which is a quite fair description of the experimental setup studied here, and with experimental results from the air/liquid interface of the capillary rising

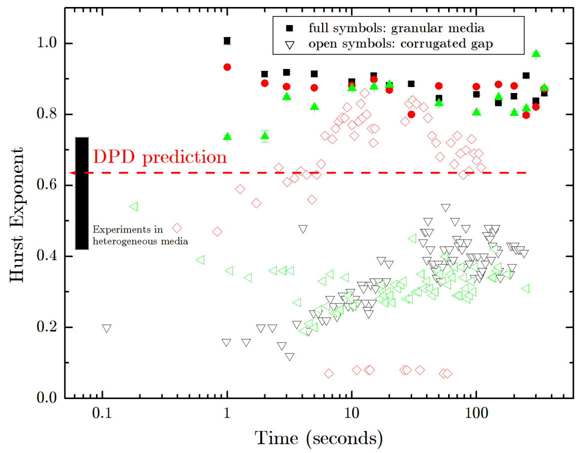

FIGURE 4. Time evolution of the Hurst exponent for the air/liquid interface in the displacement of air by water into granular media (filled symbols) or in the gap of two corrugated plates (open symbols). Three runs of each experiment is shown to facilitate the comparison. The value of H predicted from DPD model is indicated with a dashed arrow, and the range of experimental results is marked with a gray rectangle.

in a corrugated gap in modified Hele-Shaw cell. Our results leads to $H > 0.8$ which are odd both to theoretical and experimental results. The higher connected network in the granular media when compared to the corrugated case, and the different morphologies of pores and grains can be the source of the disagreement on the results.

We expect to perform experiments with different gap widths in order to verify the importance of the 3D geometry on the values of the Hurst exponent. We also intend perform a complete characterization of the granular material in order to better model the geometry of the assembly of grains, pores and channels.

ACKNOWLEDGMENTS

APFA and GHBM thanks FAPEMIG and CNPq for financial support. APFA and GC thanks CEFET and Université Joseph Fourier for make possible a 3 month invited professor grant.

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